# Samacheer Kalvi Maths

## 8th Std

## 1.3 Four Properties of Rational Numbers

### 1.3.1 (a) Addition

### (i) Closure property

The sum of any two rational numbers is always a rational number. This is called 'Closure property of addition' of rational numbers. Thus, Q is closed under addition.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b}+\frac{c}{d}$ is also a rational number.

*Illustration:* (i) $\frac{2}{9}+\frac{4}{9}=\frac{6}{9}=\frac{2}{3}$ is a rational number.

(ii) $5+\frac{1}{3}=\frac{5}{1}+\frac{1}{3}=\frac{15+1}{3}=\frac{16}{3}=5\frac{1}{3}$ is a rational number.

### (ii) Commutative property

Addition of two rational numbers is commutative.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b}+\frac{c}{d}=\frac{c}{d}+\frac{a}{b}$.

*Illustration:* For two rational numbers $\frac{1}{2}, \frac{2}{5}$ we have

$$\frac{1}{2}+\frac{2}{5} = \frac{2}{5}+\frac{1}{2}$$

| | |
|---|---|
| LHS $=\frac{1}{2}+\frac{2}{5}$ | RHS $=\frac{2}{5}+\frac{1}{2}$ |
| $=\frac{5+4}{10}=\frac{9}{10}$ | $=\frac{4+5}{10}=\frac{9}{10}$ |

$$\therefore \text{LHS} = \text{RHS}$$

$\therefore$ Commutative property is true for addition.

### (iii) Associative property

Addition of rational numbers is associative.

If $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then $\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)=\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}$.

***Illustration:*** For three rational numbers $\frac{2}{3}$, $\frac{1}{2}$ and 2, we have

$$\frac{2}{3}+\left(\frac{1}{2}+2\right) = \left(\frac{2}{3}+\frac{1}{2}\right)+2$$

$$\begin{aligned}
\text{LHS} &= \frac{2}{3}+\left(\frac{1}{2}+2\right)\\
&= \frac{2}{3}+\left(\frac{1}{2}+\frac{2}{1}\right)\\
&= \frac{2}{3}+\left(\frac{1}{2}+\frac{4}{2}\right) = \frac{2}{3}+\frac{5}{2}\\
&= \frac{4+15}{6} = \frac{19}{6} = 3\frac{1}{6}
\end{aligned}$$

$$\begin{aligned}
\text{RHS} &= \left(\frac{2}{3}+\frac{1}{2}\right)+2\\
&= \left(\frac{4}{6}+\frac{3}{6}\right)+2\\
&= \frac{7}{6}+2 = \frac{7}{6}+\frac{2}{1}\\
&= \frac{7+12}{6} = \frac{19}{6} = 3\frac{1}{6}
\end{aligned}$$

$\therefore$ LHS = RHS

$\therefore$ Associative property is true for addition.

### (iv) Additive identity

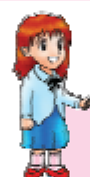
The sum of any rational number and zero is the rational number itself.

If $\frac{a}{b}$ is any rational number, then $\frac{a}{b}+0=\frac{a}{b}=0+\frac{a}{b}$.

Zero is the additive identity for rational numbers.

***Illustration:*** (i) $\frac{2}{7}+0=\frac{2}{7}=0+\frac{2}{7}$

(ii) $\left(\frac{-7}{11}\right)+0=\frac{-7}{11}=0+\left(\frac{-7}{11}\right)$



*Do you know?*

Zero is a special rational number. It can be written as $0=\frac{0}{q}$ where $q \neq 0$.

### (v) Additive inverse

$\left(\frac{-a}{b}\right)$ is the negative or additive inverse of $\frac{a}{b}$.

If $\frac{a}{b}$ is a rational number, then there exists a rational number $\left(\frac{-a}{b}\right)$ such that $\frac{a}{b}+\left(\frac{-a}{b}\right)=0$.

***Illustration:*** (i) Additive inverse of $\frac{3}{5}$ is $\frac{-3}{5}$

(ii) Additive inverse of $\frac{-3}{5}$ is $\frac{3}{5}$

(iii) Additive inverse of 0 is 0 itself.

### 1.3.1 (b) Subtraction

#### (i) Closure Property

The difference between any two rational numbers is always a rational number. Hence Q is closed under subtraction.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} - \frac{c}{d}$ is also a rational number.

***Illustration:*** (i) $\frac{4}{7} - \frac{2}{7} = \frac{2}{7}$ is a rational number.

(ii) $1 - \frac{1}{2} = \frac{2-1}{2} = \frac{1}{2}$ is a rational number.

#### (ii) Commutative Property

Subtraction of two rational numbers is not commutative.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} - \frac{c}{d} \neq \frac{c}{d} - \frac{a}{b}$.

***Illustration:*** For two rational numbers $\frac{4}{9}$ and $\frac{2}{5}$, we have

$$\frac{4}{9} - \frac{2}{5} \neq \frac{2}{5} - \frac{4}{9}$$

$$\text{LHS} = \frac{4}{9} - \frac{2}{5} = \frac{20-18}{45} = \frac{2}{45}$$

$$\text{RHS} = \frac{2}{5} - \frac{4}{9} = \frac{18-20}{45} = \frac{-2}{45}$$

$$\therefore \text{LHS} \neq \text{RHS}$$

$\therefore$ Commutative property is not true for subtraction.

**Do you know?** When two rational numbers are equal, then commutative property is true for them.

### (iii) Associative property

Subtraction of rational numbers is not associative.

If $\frac{a}{b}$ , $\frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then $\frac{a}{b}-\left(\frac{c}{d}-\frac{e}{f}\right) \neq \left(\frac{a}{b}-\frac{c}{d}\right)-\frac{e}{f}$.

***Illustration:*** For three rational numbers $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, we have

$$\frac{1}{2}-\left(\frac{1}{3}-\frac{1}{4}\right) \neq \left(\frac{1}{2}-\frac{1}{3}\right)-\frac{1}{4}$$

$$\text{LHS} = \frac{1}{2}-\left(\frac{1}{3}-\frac{1}{4}\right) = \frac{1}{2}-\left(\frac{4-3}{12}\right) = \frac{1}{2}-\left(\frac{1}{12}\right) = \frac{6-1}{12} = \frac{5}{12}$$

$$\text{RHS} = \left(\frac{1}{2}-\frac{1}{3}\right)-\frac{1}{4} = \left(\frac{3-2}{6}\right)-\frac{1}{4} = \frac{1}{6}-\frac{1}{4} = \frac{2-3}{12} = \frac{-1}{12}$$

$\therefore$ LHS $\neq$ RHS

$\therefore$ Associative property is not true for subtraction.

## 1.3.1 (c) Multiplication

### (i) Closure property

The product of two rational numbers is always a rational number. Hence Q is closed under multiplication.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b}\times\frac{c}{d} = \frac{ac}{bd}$ is also a rational number.

***Illustration:*** (i) $\frac{1}{3}\times 7 = \frac{7}{3} = 2\frac{1}{3}$ is a rational number.

(ii) $\frac{4}{3}\times\frac{5}{9} = \frac{20}{27}$ is a rational number.

### (ii) Commutative property

Multiplication of rational numbers is commutative.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b}\times\frac{c}{d} = \frac{c}{d}\times\frac{a}{b}$.

***Illustration:*** For two rational numbers $\frac{3}{5}$ and $\frac{-8}{11}$, we have

$$\frac{3}{5}\times\left(\frac{-8}{11}\right) = \left(\frac{-8}{11}\right)\times\frac{3}{5}$$

$$\text{LHS} = \frac{3}{5}\times\left(\frac{-8}{11}\right) = \frac{-24}{55} \qquad \text{RHS} = \frac{-8}{11}\times\left(\frac{3}{5}\right) = \frac{-24}{55}$$

$$\therefore \text{LHS} = \text{RHS}$$

$\therefore$ Commutative property is true for multiplication.

### (iii) Associative property

Multiplication of rational numbers is associative.

If $\frac{a}{b}$ , $\frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then $\frac{a}{b}\times\left(\frac{c}{d}\times\frac{e}{f}\right) = \left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{e}{f}$.

***Illustration:*** For three rational numbers $\frac{1}{2}$, $\left(\frac{-1}{4}\right)$ and $\frac{1}{3}$, we have

$$\frac{1}{2}\times\left(\frac{-1}{4}\times\frac{1}{3}\right) = \left(\frac{1}{2}\times\left(\frac{-1}{4}\right)\right)\times\frac{1}{3}$$

$$\text{LHS} = \frac{1}{2}\times\left(\frac{-1}{12}\right) = \frac{-1}{24} \qquad \text{RHS} = \left(\frac{-1}{8}\right)\times\frac{1}{3} = \frac{-1}{24}$$

$$\therefore \text{LHS} = \text{RHS}$$

$\therefore$ Associative property is true for multiplication.

### (iv) Multiplicative identity

The product of any rational number and 1 is the rational number itself. 'One' is the multiplicative identity for rational numbers.

If $\frac{a}{b}$ is any rational number, then $\frac{a}{b}\times 1 = \frac{a}{b} = 1\times\frac{a}{b}$.

***Illustration:*** (i) $\frac{5}{7}\times 1 = \frac{5}{7}$

(ii) $\left(\frac{-3}{8}\right)\times 1 = \frac{-3}{8}$


Is 1 the multiplicative identity for integers?

### (v) Multiplication by 0

Every rational number multiplied with 0 gives 0.

If $\frac{a}{b}$ is any rational number, then $\frac{a}{b} \times 0 = 0 = 0 \times \frac{a}{b}$.

***Illustration:*** (i) $-5 \times 0 = 0$

(ii) $\left(\frac{-7}{11}\right) \times 0 = 0$

### (vi) Multiplicative Inverse or Reciprocal

For every rational number $\frac{a}{b}$, $a \neq 0$, there exists a rational number $\frac{c}{d}$ such that $\frac{a}{b} \times \frac{c}{d} = 1$. Then $\frac{c}{d}$ is called the multiplicative inverse of $\frac{a}{b}$.

If $\frac{a}{b}$ is a rational number, then $\frac{b}{a}$ is the multiplicative inverse or reciprocal of it.

***Illustration:*** (i) The reciprocal of 2 is $\frac{1}{2}$.

(ii) The multiplicative inverse of $\left(\frac{-3}{5}\right)$ is $\left(\frac{-5}{3}\right)$.

## 1.3.1 (d) Division

### (i) Closure property

The collection of non-zero rational numbers is closed under division.

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, such that $\frac{c}{d} \neq 0$, then $\frac{a}{b} \div \frac{c}{d}$ is always a rational number.

*Illustration:* (i) $\frac{2}{3} \div \frac{1}{3} = \frac{2}{3} \times \frac{3}{1} = \frac{2}{1} = 2$ is a rational number.

(ii) $\frac{4}{5} \div \frac{3}{2} = \frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$ is a rational number.

### (ii) Commutative property

Division of rational numbers is not commutative.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} \div \frac{c}{d} \neq \frac{c}{d} \div \frac{a}{b}$

*Illustration:* For two rational numbers $\frac{4}{5}$ and $\frac{3}{8}$, we have

$$\frac{4}{5} \div \frac{3}{8} \neq \frac{3}{8} \div \frac{4}{5}$$

$$\text{LHS} = \frac{4}{5} \times \frac{8}{3} = \frac{32}{15} \qquad \text{RHS} = \frac{3}{8} \times \frac{5}{4} = \frac{15}{32}$$

$$\therefore \text{LHS} \neq \text{RHS}$$

$\therefore$ Commutative property is not true for division.

### (iii) Associative property

Division of rational numbers is not associative.

If $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then $\frac{a}{b} \div \left(\frac{c}{d} \div \frac{e}{f}\right) \neq \left(\frac{a}{b} \div \frac{c}{d}\right) \div \frac{e}{f}$.

*Illustration:* For three rational numbers $\frac{3}{4}$, 5 and $\frac{1}{2}$, we have

$$\frac{3}{4} \div \left(5 \div \frac{1}{2}\right) \neq \left(\frac{3}{4} \div 5\right) \div \frac{1}{2}$$

| LHS | RHS |
|---|---|
| $\text{LHS} = \frac{3}{4} \div \left(5 \div \frac{1}{2}\right)$ | $\text{RHS} = \left(\frac{3}{4} \div 5\right) \div \frac{1}{2}$ |
| $= \frac{3}{4} \div \left(\frac{5}{1} \times \frac{2}{1}\right)$ | $= \left(\frac{3}{4} \times \frac{1}{5}\right) \div \frac{1}{2}$ |
| $= \frac{3}{4} \div 10$ | $= \frac{3}{20} \times \frac{2}{1}$ |
| $= \frac{3}{4} \times \frac{1}{10} = \frac{3}{40}$ | $= \frac{3}{10}$ |

$$\therefore \text{LHS} \neq \text{RHS}$$

$\therefore$ Associative property is not true for division.

### 1.3.1 (e) Distributive Property

#### (i) Distributive property of multiplication over addition

Multiplication of rational numbers is distributive over addition.

If $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then $\frac{a}{b} \times \left(\frac{c}{d} + \frac{e}{f}\right) = \frac{a}{b} \times \frac{c}{d} + \frac{a}{b} \times \frac{e}{f}$.

***Illustration:*** For three rational numbers $\frac{2}{3}, \frac{4}{9}$ and $\frac{3}{5}$, we have

$$\frac{2}{3} \times \left(\frac{4}{9} + \frac{3}{5}\right) = \frac{2}{3} \times \frac{4}{9} + \frac{2}{3} \times \frac{3}{5}$$

| LHS | RHS |
|---|---|
| $\text{LHS} = \frac{2}{3} \times \left(\frac{4}{9} + \frac{3}{5}\right)$ | $\text{RHS} = \frac{2}{3} \times \frac{4}{9} + \frac{2}{3} \times \frac{3}{5}$ |
| $= \frac{2}{3} \times \left(\frac{20+27}{45}\right)$ | $= \frac{8}{27} + \frac{2}{5}$ |
| $= \frac{2}{3} \times \frac{47}{45} = \frac{94}{135}$ | $= \frac{40+54}{135} = \frac{94}{135}$ |

$$\therefore \text{LHS} = \text{RHS}$$

$\therefore$ Multiplication is distributive over addition.

#### (ii) Distributive property of multiplication over subtraction

Multiplication of rational numbers is distributive over subtraction.

If $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then $\frac{a}{b} \times \left(\frac{c}{d} - \frac{e}{f}\right) = \frac{a}{b} \times \frac{c}{d} - \frac{a}{b} \times \frac{e}{f}$.

*Illustration:* For three rational numbers $\frac{3}{7}$, $\frac{4}{5}$ and $\frac{1}{2}$, we have

$$\frac{3}{7}\times\left(\frac{4}{5}-\frac{1}{2}\right) = \frac{3}{7}\times\frac{4}{5}-\frac{3}{7}\times\frac{1}{2}$$

$$\text{LHS} = \frac{3}{7}\times\left(\frac{4}{5}-\frac{1}{2}\right) = \frac{3}{7}\times\left(\frac{8-5}{10}\right) = \frac{3}{7}\times\frac{3}{10} = \frac{9}{70}$$

$$\text{RHS} = \frac{3}{7}\times\frac{4}{5}-\frac{3}{7}\times\frac{1}{2} = \frac{12}{35}-\frac{3}{14} = \frac{24-15}{70} = \frac{9}{70}$$

$$\therefore \text{LHS} = \text{RHS}$$

$\therefore$ Multiplication is distributive over subtraction.

**B** - brackets, **O** - of, **D** - division, **M** - multiplication, **A** - addition, **S** - subtraction.

Now we will study more about brackets and operation - of.

## Brackets

Some grouping symbols are employed to indicate a preference in the order of operations. Most commonly used grouping symbols are given below.

| Grouping symbols | Names |
|---|---|
| —— | Bar bracket or Vinculum |
| ( ) | Parenthesis or common brackets |
| { } | Braces or Curly brackets |
| [ ] | Brackets or Square brackets |

## Operation - "Of "

We sometimes come across expressions like 'twice of 3', 'one - fourth of 20', 'half of 10' etc. In these expressions, 'of' means 'multiplication with'.

For example,

(i) 'twice of 3' is written as $2 \times 3$,

(ii) 'one - fourth' of 20 is written as $\frac{1}{4}\times 20$,

(iii) 'half of 10' is written as $\frac{1}{2}\times 10$.

If more than one grouping symbols are used, we first perform the operations within the innermost symbol and remove it. Next we proceed to the operations within the next innermost symbols and so on.

## 1.6 Laws of Exponents with Integral Powers

With the above definition of positive integral power of a real number, we now establish the following properties called "laws of indices" or "laws of exponents".

### (i) Product Rule

| **Law 1** | $a^m \times a^n = a^{m+n}$, where '$a$' is a real number and $m$, $n$ are positive integers |
|---|---|

***Illustration***

$\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)^{3+4} = \left(\frac{2}{3}\right)^7$ (Using the law, $a^m \times a^n = a^{m+n}$, where $a = \frac{2}{3}$, $m = 3, n = 4$)

### (ii) Quotient Rule

| **Law 2** | $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$ and $m$, $n$ are positive integers with $m > n$ |
|---|---|

***Illustration***

$\frac{6^4}{6^2} = 6^{4-2} = 6^2$ **(Using the law $\frac{a^m}{a^n} = a^{m-n}$, where a = 6, m=4, n=2)**

### (iii) Power Rule

| **Law 3** | $(a^m)^n = a^{m \times n}$, where $m$ and $n$ are positive integers |
|---|---|

***Illustration***

$(3^2)^4 = 3^2 \times 3^2 \times 3^2 \times 3^2 = 3^{2+2+2+2} = 3^8$

we can get the same result by multiplying the two powers

i.e, $(3^2)^4 = 3^{2\times 4} = 3^8$.

**Try these**

Show that $a^{(x-y)z} \times a^{(y-z)x} \times a^{(z-x)y} = 1$

### (iv) Number with zero exponent

For $m \neq o$,

$m^3 \div m^3 = m^{3-3} = m^0$ (using law 2);

Aliter:

$m^3 \div m^3 = \frac{m^3}{m^3} = \frac{m \times m \times m}{m \times m \times m} = 1$

Using these two methods, $m^3 \div m^3 = m^0 = 1$.

From the above example, we come to the **fourth law** of exponent

| **Law 4** | If '$a$' is a rational number other than "zero", then $a^0 = 1$ |
|---|---|

***Illustration***

(i) $2^0 = 1$ (ii) $\left(\frac{3}{4}\right)^0 = 1$ (iii) $25^0 = 1$ (iv) $\left(-\frac{2}{5}\right)^0 = 1$ (v) $(-100)^0 = 1$

### (v) Law of Reciprocal

The value of a number with negative exponent is calculated by converting into multiplicative inverse of the same number with positive exponent.

*Illustration*

(i) $4^{-4} = \frac{1}{4^4} = \frac{1}{4 \times 4 \times 4 \times 4} = \frac{1}{256}$

(ii) $5^{-3} = \frac{1}{5^3} = \frac{1}{5 \times 5 \times 5} = \frac{1}{125}$

(iii) $10^{-2} = \frac{1}{10^2} = \frac{1}{10 \times 10} = \frac{1}{100}$

Reciprocal of 3 is equal to $\frac{1}{3} = \frac{3^0}{3^1} = 3^{0-1} = 3^{-1}$.

Similarly, reciprocal of $6^2 = \frac{1}{6^2} = \frac{6^0}{6^2} = 6^{0-2} = 6^{-2}$

Further, reciprocal of $\left(\frac{8}{3}\right)^3$ is equal to $\frac{1}{\left(\frac{8}{3}\right)^3} = \left(\frac{8}{3}\right)^{-3}$.

From the above examples, we come to the fifth law of exponent.

| | |
|---|---|
| **Law 5** | If '$a$' is a real number and '$m$' is an integer, then $a^{-m} = \frac{1}{a^m}$ |

## (vi) Multiplying numbers with same exponents

Consider the simplifications,

(i) $4^3 \times 7^3 = (4 \times 4 \times 4) \times (7 \times 7 \times 7) = (4 \times 7) \times (4 \times 7) \times (4 \times 7)$

$= (4 \times 7)^3$

(ii) $5^{-3} \times 4^{-3} = \frac{1}{5^3} \times \frac{1}{4^3} = \left(\frac{1}{5}\right)^3 \times \left(\frac{1}{4}\right)^3$

$= \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}$

$= \left(\frac{1}{5} \times \frac{1}{4}\right) \times \left(\frac{1}{5} \times \frac{1}{4}\right) \times \left(\frac{1}{5} \times \frac{1}{4}\right) = \left(\frac{1}{20}\right)^3$

$= 20^{-3} = (5 \times 4)^{-3}$

(iii) $\left(\frac{3}{5}\right)^2 \times \left(\frac{1}{2}\right)^2 = \left(\frac{3}{5} \times \frac{3}{5}\right) \times \left(\frac{1}{2} \times \frac{1}{2}\right) = \left(\frac{3}{5} \times \frac{1}{2}\right) \times \left(\frac{3}{5} \times \frac{1}{2}\right)$

$= \left(\frac{3}{5} \times \frac{1}{2}\right)^2$

In general, for any two integers $a$ and $b$ we have

$$a^2 \times b^2 = (a \times b)^2 = (ab)^2$$

$\therefore$ We arrive at the **power of a product rule** as follows:

$$(a \times a \times a \times \ldots m \text{ times}) \times (b \times b \times b \times \ldots m \text{ times}) = ab \times ab \times ab \times \ldots m \text{ times} = (ab)^m$$

(i.e.,) $a^m \times b^m = (ab)^m$

| **Law 6** | $a^m \times b^m = (ab)^m$, where $a$, $b$ are real numbers and $m$ is an integer. |
|---|---|

*Illustration*

(i) $3^x \times 4^x = (3 \times 4)^x = 12^x$

(ii) $7^2 \times 2^2 = (7 \times 2)^2 = 14^2 = 196$

## (vii) Power of a quotient rule

Consider the simplifications,

(i) $\left(\frac{4}{3}\right)^2 = \frac{4}{3} \times \frac{4}{3} = \frac{16}{9} = \frac{4^2}{3^2}$ and

(ii) $\left(\frac{3}{5}\right)^{-2} = \frac{1}{\left(\frac{3}{5}\right)^2} = \frac{1}{\left(\frac{3^2}{5^2}\right)} = \frac{5^2}{3^2} = \left(\frac{5}{3}\right)^2 \quad \left(\because a^{-m} = \frac{1}{a^m}\right)$

$= \frac{5}{3} \times \frac{5}{3} = \frac{5 \times 5}{3 \times 3} = \frac{5^2}{3^2} = 5^2 \times \frac{1}{3^2} = 5^2 \times 3^{-2} = \frac{1}{5^{-2}} \times 3^{-2}$

$= \frac{3^{-2}}{5^{-2}}.$

Hence $\left(\frac{a}{b}\right)^2$ can be written as $\frac{a^2}{b^2}$

$$\left(\frac{a}{b}\right)^m = \left(\frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \ldots m \text{ times}\right) = \frac{a \times a \times a \ldots m \text{ times}}{b \times b \times b \times \ldots m \text{ times}}$$

$$\therefore \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

| Law 7 | $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, where $b \neq 0$, $a$ and $b$ are real numbers, $m$ is an integer |
|---|---|

***Illustration***

(i) $\left(\frac{a}{b}\right)^7 = \frac{a^7}{b^7}$    (ii) $\left(\frac{5}{3}\right)^3 = \frac{5^3}{3^3} = \frac{125}{27}$

(iii) $\left(\frac{1}{4}\right)^4 = \frac{1^4}{4^4} = \frac{1}{256}$

***Example 1.8***

Simplify: (i) $2^5 \times 2^3$  (ii) $10^9 \div 10^6$  (iii) $(x^0)^4$  (iv) $(2^3)^0$

(v) $\left(\frac{3}{2}\right)^5$  (vi) $(2^5)^2$  (vii) $(2 \times 3)^4$

(viii) If $2^p = 32$, find the value of $p$.

***Solution***

(i) $2^5 \times 2^3 = 2^{5+3} = 2^8$

(ii) $10^9 \div 10^6 = 10^{9-6} = 10^3$

(iii) $(x^0)^4 = (1)^4 = 1$  $[\because a^0 = 1]$

(iv) $(2^3)^0 = 8^0 = 1$  $[\because a^0 = 1]$

(v) $\left(\frac{3}{2}\right)^5 = \frac{3^5}{2^5} = \frac{243}{32}$

(vi) $(2^5)^2 = 2^{5\times 2} = 2^{10} = 1024$

(vii) $(2 \times 3)^4 = 6^4 = 1296$

(or) $(2 \times 3)^4 = 2^4 \times 3^4 = 16 \times 81 = 1296$

(viii) Given : $2^p = 32$

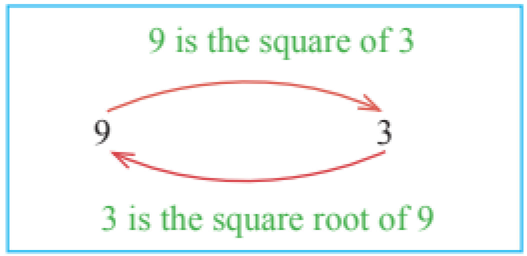
$2^p = 2^5$

Therefore $p = 5$ (Here the base on both sides are equal.)

| | |
|---|---|
| 2 | 32 |
| 2 | 16 |
| 2 | 8 |
| 2 | 4 |
| 2 | 2 |
| | 1 |

## 1.7.2 Square roots

### Definition




When a number is multiplied by itself, the product is called the square of that number. The number itself is called the **square root** of the product.

For example:

(i) $3 \times 3 = 3^2 = 9$

(ii) $(-3) \times (-3) = (-3)^2 = 9$

Here 3 and $(-3)$ are the square roots of 9.

**The symbol used for square root is $\sqrt{\phantom{x}}$ .**

$\therefore \sqrt{9} = \pm 3$ ( read as plus or minus 3 )

Considering only the positive root, we have $\sqrt{9} = 3$

> **Note:** We write the square root of $x$ as $\sqrt{x}$ or $x^{\frac{1}{2}}$.
> Hence, $\sqrt{4} = (4)^{\frac{1}{2}}$ and $\sqrt{100} = (100)^{\frac{1}{2}}$

To find a square root of a number, we have the following two methods.

**(i) Factorization Method**
**(ii) Long Division Method**

### (i) Factorization Method

The square root of a perfect square number can be found by finding the prime factors of the number and grouping them in pairs.

*Example 1.17*

Find the square root of 64

*Solution*

$$64 = \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{2 \times 2} = 2^2 \times 2^2 \times 2^2$$

$$\sqrt{64} = \sqrt{2^2 \times 2^2 \times 2^2} = 2 \times 2 \times 2 = 8$$

$$\sqrt{64} = 8$$

Prime factorization

| | |
|---|---|
| 2 | 64 |
| 2 | 32 |
| 2 | 16 |
| 2 | 8 |
| 2 | 4 |
| 2 | 2 |
| | 1 |

## (ii) Long division method

In case of large numbers, factors can not be found easily. Hence we may use another method, known as **Long division method**.

Using this method, we can also find square roots of decimal numbers. This method is explained in the following worked examples.

*Example 1.23*

Find the square root of 529 using long division method.

*Solution*

**Step 1** : We write 529 as $5\ \overline{29}$ by grouping the numbers in pairs, starting from the right end. (i.e. from the unit's place ).

**Step 2** : Find the number whose square is less than (or equal to) 5. Here it is 2.

$$\begin{array}{r|l} 2 & 5\ \overline{29} \end{array}$$

**Step 3** : Put '2' on the top, and also write 2 as a divisor as shown.

$$\begin{array}{r|l} & 2 \\ \hline 2 & 5\ \overline{29} \\ & 4 \\ \hline & 1 \end{array}$$

**Step 4** : Multiply 2 on the top with the divisor 2 and write 4 under 5 and subtract. The remainder is 1.

**Step 5** : Bring down the pair 29 by the side of the remainder 1, yielding 129.

$$\begin{array}{r|l} & 2 \\ \hline 2 & 5\ \overline{29} \\ & 4\ \downarrow \\ \hline & 1\ 29 \end{array}$$

**Step 6** : Double 2 and take the resulting number 4. Find that number '$n$' such that $4n \times n$ is just less than or equal to 129.

For example : $42 \times 2 = 84$; and $43 \times 3 = 129$ and so $n = 3$.

$$\begin{array}{r|l} & 2\ \ 3 \\ \hline 2 & 5\ \overline{29} \\ & 4\ \downarrow \\ \hline 43 & 1\ 29 \\ & 1\ 29 \\ \hline & \quad 0 \end{array}$$

**Step 7** : Write 43 as the next divisor and put 3 on the top along with 2. Write the product $43 \times 3 = 129$ under 129 and subtract. Since the remainder is '0', the division is complete.

Hence $\sqrt{529} = 23$.

## *Example 1.25*

Find the square root of 6.0516

### *Solution*

We write the number as $6\,.\,\overline{05}\,\overline{16}$. Since the number of digits in the integral part is 1, the square root will have 1 digit in its integral part. We follow the same procedure that we usually use to find the square root of 60516

$$
\begin{array}{r|r}
 & 2.\,4\;\;6 \\
\hline
2 & 6.\overline{05}\;\overline{16} \\
 & 4\phantom{.05\;16} \\
\hline
44 & 2\;05\phantom{\;16} \\
 & 1\;76\phantom{\;16} \\
\hline
486 & 29\,16 \\
 & 29\,16 \\
\hline
 & 0
\end{array}
$$

From the above working, we get $\sqrt{6.0516} = 2.46$.

## *Example 1.31*

Find the square root of 3 correct to two places of decimal.

### *Solution*

$$
\begin{array}{r|r}
 & 1.\;\;7\;\;3\;\;2 \\
\hline
1 & 3.\;\overline{00}\;\overline{00}\;\overline{00} \\
 & 1\phantom{.\;00\;00\;00} \\
\hline
27 & 2\;\;00\phantom{\;00\;00} \\
 & 1\;\;89\phantom{\;00\;00} \\
\hline
343 & 11\,00\phantom{\;00} \\
 & 10\,29\phantom{\;00} \\
\hline
3462 & 71\;\;00 \\
 & 69\;\;24 \\
\hline
 & 1\;\;76
\end{array}
$$

Since we need the answer correct to two places of decimal, we shall first find the square root up to three places of decimal. For this purpose we must add 6 ( that is three pairs of ) zeros to the right of the decimal point.

$\therefore \sqrt{3}$ = 1.732 up to three places of decimal.

$\sqrt{3}$ = 1.73 correct to two places of decimal.

*Example 1.32*

Find the square root of $10\frac{2}{3}$ correct to two places of decimal.

*Solution*

$10\frac{2}{3} = \frac{32}{3} = 10.66\ 66\ 66$ ........

| | |
|---|---|
| | 3. 2 6 5 |
| 3 | 10. $\overline{66}$ $\overline{66}$ $\overline{67}$ |
| | 9 |
| 62 | 1 66 |
| | 1 24 |
| 646 | 42 66 |
| | 38 76 |
| 6525 | 3 90 67 |
| | 3 26 25 |
| | 64 42 |

In order to find the square root correct to two places of decimal, we have to find the square root up to three places. Therefore we have to convert $\frac{2}{3}$ as a decimal correct to six places.

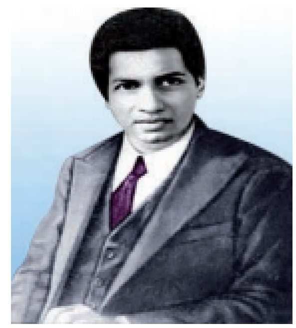
$$\sqrt{10\frac{2}{3}} = 3.265 \text{ (approximately)}$$

$$= 3.27 \text{ (correct to two places of decimal)}$$

## 1.7.3 Cubes

### Introduction

This is an incident about one of the greatest mathematical geniuses S. Ramanujan. Once mathematician Prof. G.H. Hardy came to visit him in a taxi whose taxi number was 1729. While talking to Ramanujan, Hardy described that the number 1729 was a dull number. Ramanujan quickly pointed out that 1729 was indeed an interesting number. He said, it is the smallest number that can be expressed as a sum of two cubes in two different ways.



**Srinivasa Ramanujan**
(1887 -1920)

Ramanujan, an Indian Mathematician who was born in Erode contributed the theory of numbers which brought him worldwide acclamation. During his short life time, he independently compiled nearly 3900 results.

ie., $1729 = 1728 + 1 = 12^3 + 1^3$

and $1729 = 1000 + 729 = 10^3 + 9^3$

1729 is known as the Ramanujan number.

There are many other interesting patterns of cubes, cube roots and the facts related to them.

## Cubes

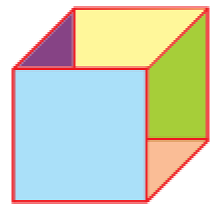
*Do you know?*

1729 is the smallest Ramanujan Number. There are an infinitely many such numbers. Few are 4104 (2, 16 ; 9, 15), 13832 (18, 20 ; 2, 24).

We know that the word 'Cube' is used in geometry. A cube is a solid figure which has all its sides are equal.

If the side of a cube in the adjoining figure is '$a$' units

then its volume is given by $a \times a \times a = a^3$ cubic units.



Here $a^3$ is called **"a cubed"** or **"$a$ raised to the power three"** or **"$a$ to the power 3"**.

Now, consider the number 1, 8, 27, 64, 125, $\cdots$

These are called **perfect cubes** or **cube numbers**.

Each of them is obtained when a number is multiplied by itself three times.

Examples: $1 \times 1 \times 1 = 1^3$, $2 \times 2 \times 2 = 2^3$, $3 \times 3 \times 3 = 3^3$, $5 \times 5 \times 5 = 5^3$

### *Example 1.33*

Find the value of the following :

(i) $15^3$ (ii) $(-4)^3$ (iii) $(1.2)^3$ (iv) $\left(\frac{-3}{4}\right)^3$

### *Solution*

(i) $15^3 = 15 \times 15 \times 15 = 3375$

(ii) $(-4)^3 = (-4) \times (-4) \times (-4) = -64$

(iii) $(1.2)^3 = 1.2 \times 1.2 \times 1.2 = 1.728$

(iv) $\left(\frac{-3}{4}\right)^3 = \frac{(-3) \times (-3) \times (-3)}{4 \times 4 \times 4} = \frac{-27}{64}$

Observe the question (ii) Here $(-4)^3 = -64$.

**Note:** When a negative number is multiplied by itself an even number of times, the product is positive. But when it is multiplied by itself an odd number of times, the product is also negative. ie, $(-1)^n = \begin{cases} -1 & \text{if n is odd} \\ +1 & \text{if n is even} \end{cases}$

**Adding consecutive odd numbers**

Observe the following pattern of sums of odd numbers.

$$1 = 1 = 1^3$$

Next 2 odd numbers, $3 + 5 = 8 = 2^3$

Next 3 odd numbers, $7 + 9 + 11 = 27 = 3^3$

Next 4 odd numbers, $13 + 15 + 17 + 19 = 64 = 4^3$

Next 5 odd numbers, $21 + 23 + 25 + 27 + 29 = 125 = 5^3$

Is it not interesting?

### *Example 1.37*

Find the cube root of 512.

***Solution***

$$\begin{aligned}\sqrt[3]{512} &= (512)^{\frac{1}{3}}\\ &= ((2\times2\times2)\times(2\times2\times2)\times(2\times2\times2))^{\frac{1}{3}}\\ &= (2^3\times2^3\times2^3)^{\frac{1}{3}}\\ &= (2^9)^{\frac{1}{3}} = 2^3\\ \sqrt[3]{512} &= 8.\end{aligned}$$

Prime factorization

$$\begin{array}{r|l} 2 & 512 \\ \hline 2 & 256 \\ \hline 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

### *Example 1.38*

Find the cube root of $27 \times 64$

***Solution***

Resolving 27 and 64 into prime factors, we get

$$\sqrt[3]{27} = (3\times3\times3)^{\frac{1}{3}} = (3^3)^{\frac{1}{3}}$$

$$\sqrt[3]{27} = 3$$

$$\begin{aligned}\sqrt[3]{64} &= (2\times2\times2\times2\times2\times2)^{\frac{1}{3}}\\ &= (2^6)^{\frac{1}{3}} = 2^2 = 4\\ \sqrt[3]{64} &= 4\\ \sqrt[3]{27\times64} &= \sqrt[3]{27}\times\sqrt[3]{64}\\ &= 3\times4\\ \sqrt[3]{27\times64} &= 12\end{aligned}$$

Prime factorization

$$\begin{array}{r|l} 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

Prime factorization

$$\begin{array}{r|l} 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

The shape of each of these objects is a 'circle'.

### (iii) Circle

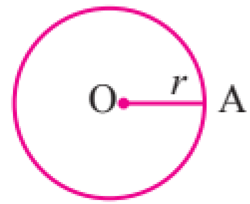


Fig. 2.3

Let 'O' be the centre of a circle with radius 'r' units $(\overline{OA})$.

Area of a circle, $A = \pi r^2$ sq.units.

Perimeter or circumference of a circle,

$$P = 2\pi r \text{ units},$$

where $\pi \simeq \frac{22}{7}$ or $3.14$.

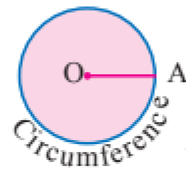


Fig. 2.4

**Note:** The central angle of a circle is 360°.

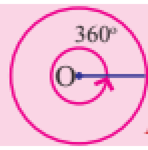


Fig. 2.5

## 2.2 Semi circles and Quadrants

### 2.2.1 Semicircle

Have you ever noticed the sky during night time after 7 days of new moon day or full moon day?

What will be the shape of the moon?

It looks like the shape of Fig. 2.6.

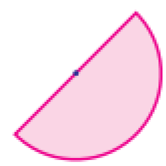


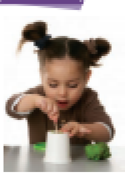
Fig. 2.6

How do you call this?

This is called a semicircle. [Half part of a circle]

The two equal parts of a circle divided by its diameter are called semicircles.

**Activity**



How will you get a semicircle from a circle?

Take a cardboard of circular shape and cut it through its diameter $\overline{AB}$.

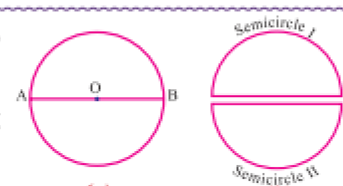


Fig. 2.7

**Note:** The central angle of the semicircle is 180°.

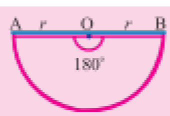


Fig. 2.8

### (a) Perimeter of a semicircle

$$\text{Perimeter, P} = \frac{1}{2} \times (\text{circumference of a circle}) + 2 \times r \text{ units}$$

$$= \frac{1}{2} \times 2\pi r + 2r$$

$$\text{P} = \pi r + 2r = (\pi + 2)\, r \text{ units}$$

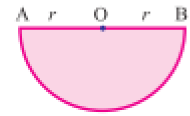


*Fig. 2.9*

### (b) Area of a semicircle

$$\text{Area, A} = \frac{1}{2} \times (\text{Area of a circle})$$

$$= \frac{1}{2} \times \pi r^2$$

$$\text{A} = \frac{\pi r^2}{2} \text{ sq. units.}$$

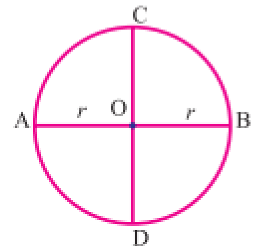


*Fig. 2.10*

### 4.2.2 Quadrant of a circle

Cut the circle through two of its perpendicular diameters. We get four equal parts of the circle. Each part is called a quadrant of the circle. We get four quadrants OCA, OAD, ODB and OBC while cutting the circle as shown in the Fig. 2.11.

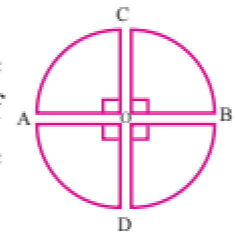


*Fig. 2.11*

**Note:** The central angle of the quadrant is 90°.

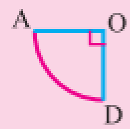


*Fig. 2.12*

### (a) Perimeter of a quadrant

$$\text{Perimeter, P} = \frac{1}{4} \times (\text{circumference of a circle}) + 2r \text{ units}$$

$$= \frac{1}{4} \times 2\pi r + 2r$$

$$\text{P} = \frac{\pi r}{2} + 2r = \left(\frac{\pi}{2} + 2\right) r \text{ units}$$

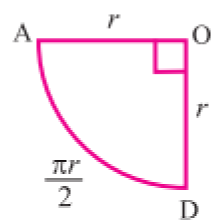
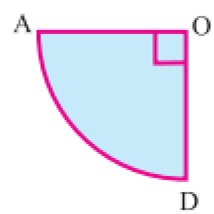


*Fig. 2.13*

### (b) Area of a quadrant

$$\text{Area, A} = \frac{1}{4} \times (\text{Area of a circle})$$

$$\text{A} = \frac{1}{4} \times \pi r^2 \text{ sq.units}$$



### *Example 2.1*

*Fig. 2.14*

Find the perimeter and area of a semicircle whose radius is 14 cm.

*Solution*


*Fig. 2.15*

**Given:** Radius of a semicircle, $r = 14$ cm

Perimeter of a semicircle, $P = (\pi + 2)\, r$ units

$$\therefore P = \left(\frac{22}{7} + 2\right) \times 14$$

$$= \left(\frac{22+14}{7}\right) \times 14 = \frac{36}{7} \times 14 = 72$$

Perimeter of the semicircle $= 72$ cm.

Area of a semicircle, $A = \frac{\pi r^2}{2}$ sq. units

$$\therefore A = \frac{22}{7} \times \frac{14 \times 14}{2} = 308 \text{ cm}^2.$$

### *Example 2.2*

The radius of a circle is 21 cm. Find the perimeter and area of a quadrant of the circle.

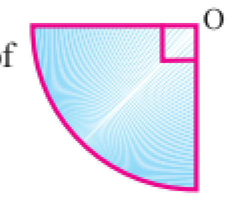



*Fig. 2.16*

*Solution*

**Given:** Radius of a circle, $r = 21$ cm

Perimeter of a quadrant, $P = \left(\frac{\pi}{2} + 2\right) r$ units

$$= \left(\frac{22}{7 \times 2} + 2\right) \times 21 = \left(\frac{22}{14} + 2\right) \times 21$$

$$P = \left(\frac{22+28}{14}\right) \times 21 = \frac{50}{14} \times 21$$

$$= 75 \text{ cm}.$$

Area of a quadrant, $A = \frac{\pi r^2}{4}$ sq. units

$$A = \frac{22}{7} \times \frac{21 \times 21}{4}$$

$$= 346.5 \text{ cm}^2.$$

### Example 2.3

The diameter of a semicircular grass plot is 14 m. Find the cost of fencing the plot at ₹ 10 per metre .

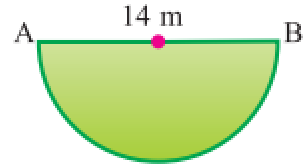



Fig. 2.17

**Solution**

**Given:** Diameter, $d = 14$ m.

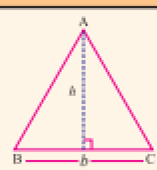
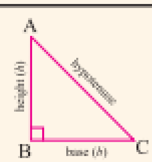
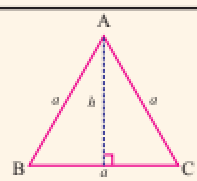
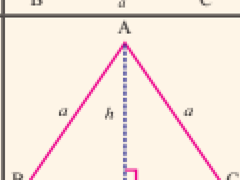
$\therefore$ Radius of the plot, $r = \frac{14}{2} = 7$m.

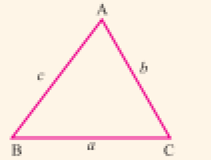
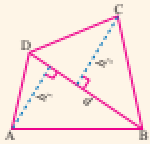
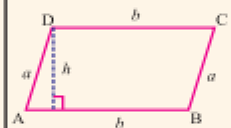
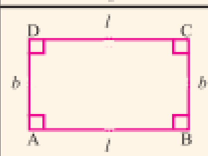
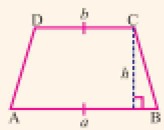
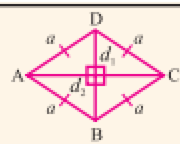
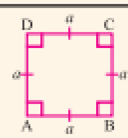
To fence the semicircular plot, we have to find the perimeter of it.

$$\text{Perimeter of a semicircle, P} = (\pi + 2) \times \text{r units}$$
$$= \left(\frac{22}{7} + 2\right) \times 7$$
$$= \left(\frac{22 + 14}{7}\right) \times 7$$
$$\text{P} = 36 \text{ m}$$

Cost of fencing the plot for 1 metre = ₹ 10

$\therefore$ Cost of fencing the plot for 36 metres = 36 × 10 = ₹ 360.

| No. | Name of the Figure | Figure | Area (A) (sq. units) | Perimeter (P) (units) |
|---|---|---|---|---|
| 1. | Triangle |  | $\frac{1}{2} \times b \times h$ | AB + BC + CA |
| 2. | Right triangle |  | $\frac{1}{2} \times b \times h$ | (base + height + hypotenuse) |
| 3. | Equilateral triangle |  | $\frac{\sqrt{3}}{4}a^2$ where ($\sqrt{3} \simeq 1.732$) | AB+BC+CA = $3a$ ; Altitude, $h = \frac{\sqrt{3}}{2}a$ units |
| 4. | Isosceles triangle |  | $h \times \sqrt{a^2 - h^2}$ | $2a + 2\sqrt{a^2 - h^2}$ |

| | | | | |
|---|---|---|---|---|
| 5. | Scalene triangle |  | $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ | AB + BC + CA $= (a+b+c)$ |
| 6. | Quadrilateral |  | $\frac{1}{2} \times d \times (h_1 + h_2)$ | AB + BC + CD + DA |
| 7. | Parallelogram |  | $b \times h$ | $2 \times (a+b)$ |
| 8. | Rectangle |  | $l \times b$ | $2 \times (l+b)$ |
| 9. | Trapezium |  | $\frac{1}{2} \times h \times (a+b)$ | AB + BC + CD + DA |
| 10. | Rhombus |  | $\frac{1}{2} \times d_1 \times d_2$ where $d_1, d_2$ are diagonals | $4a$ |
| 11. | Square |  | $a^2$ | $4a$ |

*Example 2.5*

Find the perimeter and area of the following combined figures.

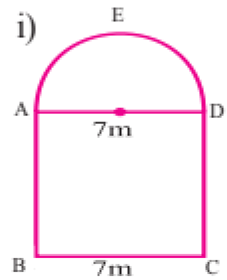
i) 


*Fig. 2.26*

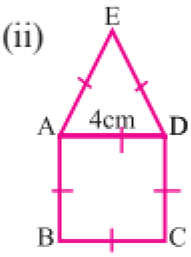
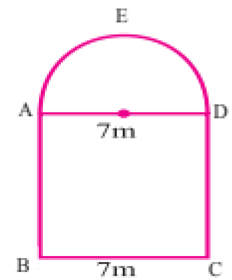
(ii) 


*Fig. 2.27*

**Solution**

(i) It is a combined figure made up of a square ABCD and a semicircle DEA. Here, are $\overparen{DEA}$ is half the circumference of a circle whose diameter is AD.




**Given:** Side of a square $= 7$ m

$\therefore$ Diameter of a semicircle $= 7$ m

$\therefore$ Radius of a semicircle, $r = \frac{7}{2}$ m

Perimeter of the combined figure $= \overline{AB}+\overline{BC}+\overline{CD}+\overparen{DEA}$

$$P = 7 + 7 + 7 + \frac{1}{2} \times \text{(circumference of a circle)}$$

$$= 21 + \frac{1}{2} \times 2\pi r = 21 + \frac{22}{7} \times \frac{7}{2}$$

$$P = 21 + 11 = 32 \text{ m}$$

. Perimeter of the combined figure $= 32$ m.

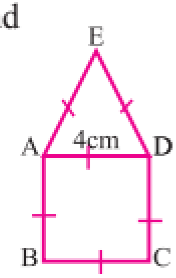
Area of the combined figure $=$ Area of a semicircle + Area of a square

$$A = \frac{\pi r^2}{2} + a^2$$

$$= \frac{22}{7 \times 2} \times \frac{7 \times 7}{2 \times 2} + 7^2 = \frac{77}{4} + 49$$

$\therefore$ Area of the given combined figure $= 19.25 + 49 = 68.25 \text{ m}^2$.

(ii) The given combined figure is made up of a square ABCD and an equilateral triangle DEA.




**Given:** Side of a square = 4 cm

$\therefore$ Perimeter of the combined figure = AB + BC + CD + DE + EA

$= 4 + 4 + 4 + 4 + 4 = 20$ cm

$\therefore$ Perimeter of the combined figure = 20 cm.

Area of the given combined figure = Area of a square + Area of an equilateral triangle

$$= a^2 + \frac{\sqrt{3}}{4}a^2 \qquad \sqrt{3} = 1.732$$

$$= 4 \times 4 + \frac{\sqrt{3}}{4} \times 4 \times 4$$

$$= 16 + 1.732 \times 4$$

Area of the given combined figure $= 16 + 6.928 = 22.928$

Area of the given figure $\simeq 22.93$ cm$^2$.

*Example 2.6*

Find the perimeter and area of the shaded portion

(i)

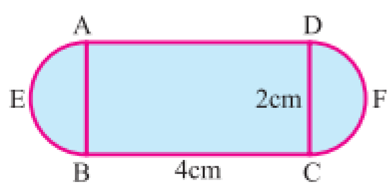



*Fig. 2.28*

(ii)

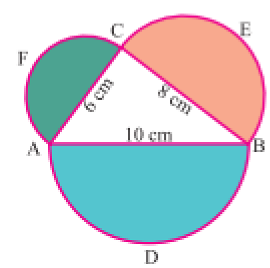
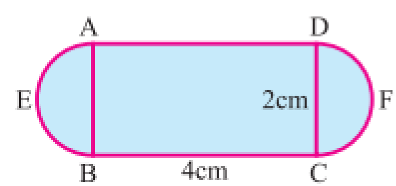



*Fig. 2.29*

**Solution**

(i) The given figure is a combination of a rectangle ABCD and two semicircles AEB and DFC of equal area.




**Given:** Length of the rectangle, $l = 4$ cm

Breadth of the rectangle, $b$ = 2 cm

Diameter of a semicircle = 2 cm

$\therefore$ Radius of a semicircle, $r = \frac{2}{2} = 1$ cm

$\therefore$ Perimeter of the given figure $= AD + BC + \widehat{AEB} + \widehat{DFC}$

$= 4 + 4 + 2 \times \frac{1}{2} \times$ (circumference of a circle)

$$= 4+4+2\times\frac{1}{2}\times(\text{circumference of a circle})$$
$$= 8+2\times\frac{1}{2}\times 2\pi r$$
$$= 8+2\times\frac{22}{7}\times 1$$
$$= 8+2\times 3.14$$
$$= 8+6.28$$

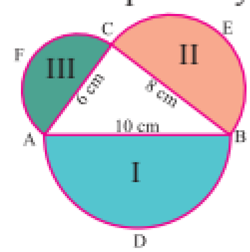
$\therefore$ Perimeter of the given figure = 14.28 cm.

Area of the given figure = Area of a rectangle ABCD + 2 × Area of a semicircle

$$= l\times b+2\times\frac{\pi r^2}{2}$$
$$= 4\times 2+2\times\frac{22\times 1\times 1}{7\times 2}$$

$\therefore$ Total area $= 8+3.14 = 11.14\text{ cm}^2$.

(ii) Let ADB, BEC and CFA be the three semicircles I, II and III respectively.




**Given:**

Radius of a semicircle I, $r_1 = \frac{10}{2} = 5$ cm

Radius of a semicircle II, $r_2 = \frac{8}{2} = 4$ cm

Radius of a semicircle III, $r_3 = \frac{6}{2} = 3$ cm

Perimeter of the shaded portion = Perimeter of a semicircle I + Perimeter of a semicircle II + Perimeter of a semicircle III

$$= (\pi + 2)\times 5 + (\pi + 2)\times 4 + (\pi + 2)\times 3$$

$$= (\pi + 2)(5 + 4 + 3) = (\pi + 2)\times 12$$

$$= \left(\frac{22 + 14}{7}\right)\times 12 = \frac{36}{7}\times 12 = 61.714$$

Perimeter of the shaded portion $\simeq$ 61.71cm.

Area of the shaded portion, A = Area of a semicircle I + Area of a semicircle II + Area of a semicircle III

$$A = \frac{\pi r_1^2}{2} + \frac{\pi r_2^2}{2} + \frac{\pi r_3^2}{2}$$

$$= \frac{22}{7\times 2}\times 5\times 5 + \frac{22}{7\times 2}\times 4\times 4 + \frac{22}{7\times 2}\times 3\times 3$$

$$A = \frac{275}{7} + \frac{176}{7} + \frac{99}{7} = \frac{550}{7} = 78.571\ \text{cm}^2$$

Area of the shaded portion $\simeq 78.57\ \text{cm}^2$

In this example we observe that,

**Area of semicircle BEC + Area of semicircle CFA = Area of semicircle ADB**

*Example 2.7*

A horse is tethered to one corner of a rectangular field of dimensions 70 m by 52 m by a rope 28 m long for grazing. How much area can the horse graze inside? How much area is left ungrazed?

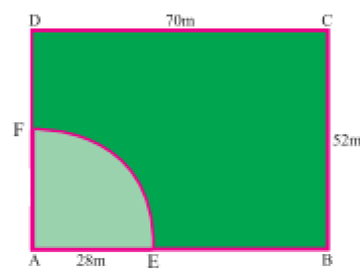


*Fig. 2.30*

*Solution*

Length of the rectangle, $l$ = 70 m

Breadth of the rectangle, $b$ = 52 m

Length of the rope = 28 m

Shaded portion AEF indicates the area in which the horse can graze. Clearly, it is the area of a quadrant of a circle of radius, $r = 28$ m

Area of the quadrant AEF $= \frac{1}{4}\times \pi r^2$ sq. units

$$= \frac{1}{4}\times\frac{22}{7}\times 28\times 28 = 616\ \text{m}^2$$

$\therefore$ Grazing Area = 616 m$^2$.

$\therefore$ Grazing Area $= 616 \text{ m}^2$.

Area left ungrazed = Area of the rectangle ABCD – Area of the quadrant AEF

Area of the rectangle ABCD $= l \times b$ sq. units

$= 70 \times 52 = 3640 \text{ m}^2$

$\therefore$ Area left ungrazed $= 3640 - 616 = 3024 \text{ m}^2$.

*Example 2.8*

In the given figure, ABCD is a square of side 14 cm. Find the area of the shaded portion.

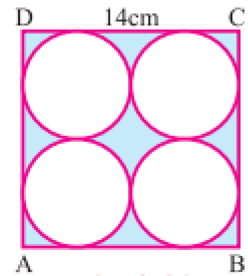


*Fig. 2.31*

*Solution*

Side of a square, $a = 14$ cm

Radius of each circle, $r = \frac{7}{2}$ cm

Area of the shaded portion = Area of a square – 4 × Area of a circle

$= a^2 - 4(\pi r^2)$

$= 14 \times 14 - 4 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= 196 - 154$

$\therefore$ Area of the shaded portion $= 42 \text{ cm}^2$.

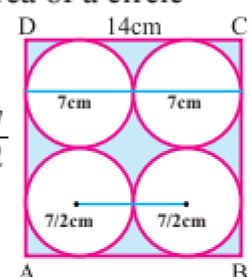


*Fig. 2.32*

### *Example 2.9*

A copper wire is in the form of a circle with radius 35 cm. It is bent into a square. Determine the side of the square.

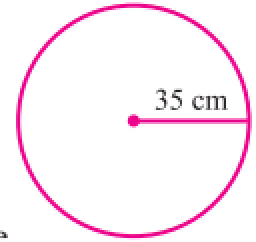



*Fig. 2.33*

***Solution***

**Given:** Radius of a circle, $r$ = 35 cm.

Since the same wire is bent into the form of a square,

$$\text{Perimeter of the circle} = \text{Perimeter of the square}$$

$$\text{Perimeter of the circle} = 2\pi r \text{ units}$$

$$= 2 \times \frac{22}{7} \times 35 \text{ cm}$$

$$P = 220 \text{ cm.}$$

Let '$a$' be the side of a square.

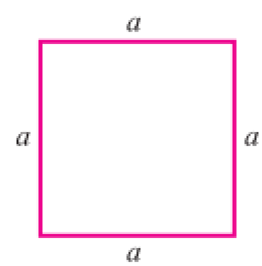



*Fig. 2.34*

$$\text{Perimeter of a square} = 4a \text{ units}$$

$$4a = 220$$

$$a = 55 \text{ cm}$$

$$\therefore \text{Side of the square} = 55 \text{ cm.}$$

*Example 2.10*

Four equal circles are described about four corners of a square so that each touches two of the others as shown in the Fig. 2.35. Find the area of the shaded portion, each side of the square measuring 28 cm.

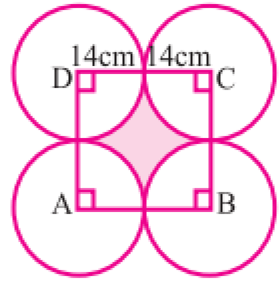



*Fig. 2.35*

*Solution*

Let ABCD be the given square of side $a$.

$$\therefore a = 28\text{cm}$$

$$\therefore \text{Radius of each circle, } r = \frac{28}{2} = 14 \text{ cm}$$

$$\text{Area of the shaded portion} = \text{Area of a square} - 4 \times \text{Area of a quadrant}$$

$$= a^2 - 4 \times \frac{1}{4} \times \pi r^2$$

$$= 28 \times 28 - 4 \times \frac{1}{4} \times \frac{22}{7} \times 14 \times 14$$

$$= 784 - 616$$

$$\therefore \text{Area of the shaded portion} = 168 \text{ cm}^2.$$

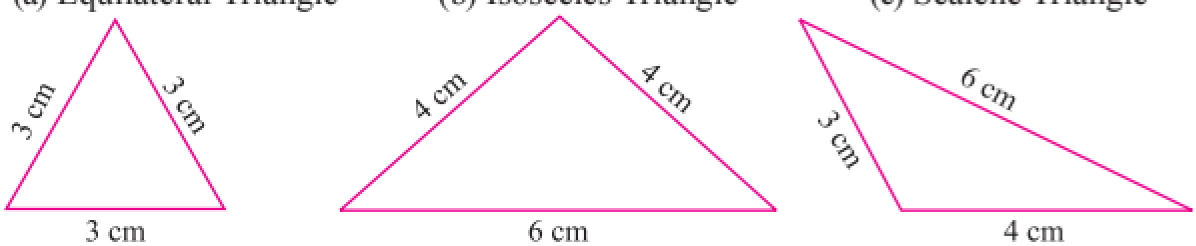
- The central angle of a circle is 360°.
- Perimeter of a semicircle = $(\pi + 2) \times r$ units.
- Area of a semicircle = $\frac{\pi r^2}{2}$ sq . units.
- The central angle of a semicircle is 180°.
- Perimeter of a quadrant = $\left(\frac{\pi}{2} + 2\right) \times r$ units.
- Area of a quadrant = $\frac{\pi r^2}{4}$ sq . units.
- The central angle of a quadrant is 90°.
- Perimeter of a combined figure is length of its boundary.
- A polygon is a closed plane figure formed by 'n' line segments.
- Regular polygons are polygons in which all the sides and angles are equal.
- Irregular polygons are combination of plane figures.

### 3.2.1. Kinds of Triangles

Triangles can be classified into two types based on sides and angles.

**Based on sides:**

(a) Equilateral Triangle (b) Isosceles Triangle (c) Scalene Triangle



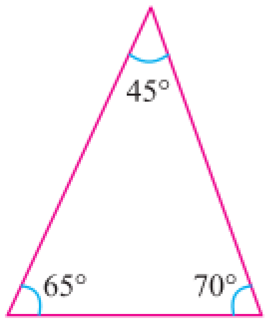

Three sides are equal

Two sides are equal

All sides are different

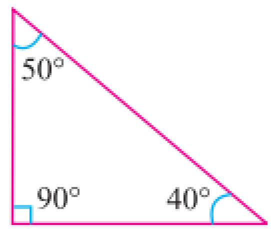
**Based on angles:**

(d) Acute Angled Triangle



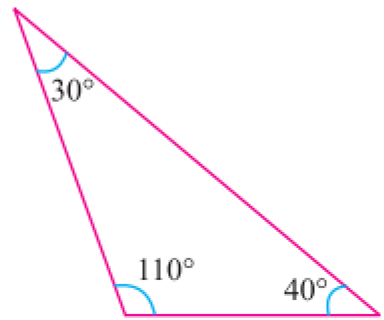

Three acute angles

(e) Right Angled Triangle




One right angle

(f) Obtuse Angled Triangle




One obtuse angle

***Example 3.1***

In $\Delta ABC$, $\angle A = 75^\circ$, $\angle B = 65^\circ$ find $\angle C$.

***Solution***

We know that in $\Delta ABC$,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$75^\circ + 65^\circ + \angle C = 180^\circ$$

$$140^\circ + \angle C = 180^\circ$$

$$\angle C = 180^\circ - 140^\circ$$

$$\therefore \angle C = 40^\circ.$$

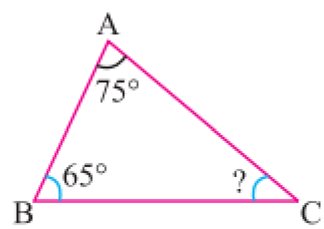



*Fig. 3.9*

### *Example 3.2*

In $\Delta ABC$, given that $\angle A = 70^\circ$ and $AB = AC$. Find the other angles of $\Delta$ ABC.

**Solution**

Let $\angle B = x^\circ$ and $\angle C = y^\circ$.

Given that $\Delta ABC$ is an isosceles triangle.

$$AC = AB$$

$$\angle B = \angle C \quad \text{[Angles opposite to equal sides are equal]}$$

$$x^\circ = y^\circ$$

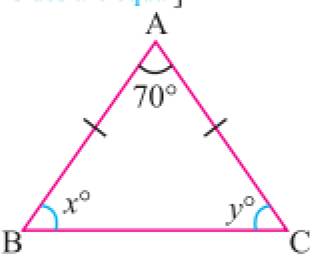



*Fig. 3.10*

$$\text{In } \Delta ABC, \angle A + \angle B + \angle C = 180^\circ$$

$$70^\circ + x^\circ + y^\circ = 180^\circ$$

$$70^\circ + x^\circ + x^\circ = 180^\circ \quad [\because x^\circ = y^\circ]$$

$$2x^\circ = 180^\circ - 70^\circ$$

$$2x^\circ = 110^\circ$$

$$x^\circ = \frac{110^\circ}{2} = 55^\circ.$$ Hence $\angle B = 55^\circ$ and $\angle C = 55^\circ$.

### *Example 3.3*

The measures of the angles of a triangle are in the ratio 5 : 4 : 3. Find the angles of the triangle.

**Solution**

Given that in a $\Delta ABC$, $\angle A : \angle B : \angle C = 5 : 4 : 3$.

Let the angles of the given triangle be $5x^\circ$, $4x^\circ$ and $3x^\circ$.

We know that the sum of the angles of a triangle is $180^\circ$.

$$5x^\circ + 4x^\circ + 3x^\circ = 180^\circ \Rightarrow 12x^\circ = 180^\circ$$

$$x^\circ = \frac{180^\circ}{12} = 15^\circ$$

So, the angles of the triangle are $75^\circ$, $60^\circ$ and $45^\circ$.

## Example 3.4

Find the angles of the triangle ABC, given in Fig.3.11.

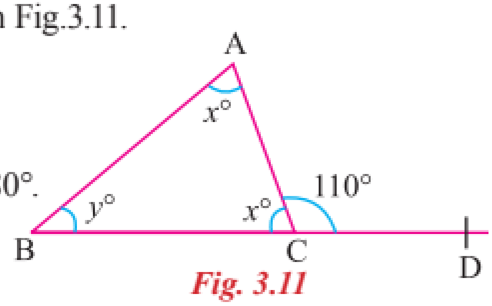



*Fig. 3.11*

### Solution

BD is a straight line.

We know that angle in the line segment is 180°.

$$x^\circ + 110^\circ = 180^\circ$$
$$x^\circ = 180^\circ - 110^\circ$$
$$x^\circ = 70^\circ$$

We know that the exterior angle is equal to the sum of the two interior opposite angles.

$$x^\circ + y^\circ = 110^\circ$$
$$70^\circ + y^\circ = 110^\circ$$
$$y^\circ = 110^\circ - 70^\circ = 40^\circ$$

Hence, $x^\circ = 70^\circ$

and $y^\circ = 40^\circ$.

## Example 1.3

(i) Subtract $5xy$ from $8xy$ (ii) Subtract $3c + 7d^2$ from $5c - d^2$

(iii) Subtract $2x^2 + 2y^2 - 6$ from $3x^2 - 7y^2 + 9$

### Solution

(i) Subtract $5xy$ from $8xy$.

The first step is to place them as

$$\begin{array}{r} 8xy \\ -5xy \\ \hline 3xy \\ \hline \end{array}$$

(The two terms $8xy$, $-5xy$ are like terms)

$\therefore 8xy - 5xy = 3xy$

(ii) Subtract $3c + 7d^2$ from $5c - d^2$

### Solution

$$\begin{array}{r} 5c - d^2 \\ -(3c + 7d^2) \\ \hline \\ \hline \end{array} \quad \text{(or)} \quad \begin{array}{r} 5c - d^2 \\ -3c - 7d^2 \\ \hline 2c - 8d^2 \\ \hline \end{array}$$

Often, we do this as →

$$\begin{array}{r} 5c - d^2 \\ 3c + 7d^2 \\ - \quad - \\ \hline 2c - 8d^2 \\ \hline \end{array}$$

Alternatively, this can also be done as :

$$\begin{aligned}(5c - d^2) - (3c + 7d^2) &= 5c - d^2 - 3c - 7d^2 \\ &= (5c - 3c) + (-d^2 - 7d^2) \\ &= 2c + (-8d^2) \\ &= 2c - 8d^2\end{aligned}$$

(iii) Subtract $2x^2 + 2y^2 - 6$ from $3x^2 - 7y^2 + 9$

*Solution*

$$\begin{array}{r} 3x^2 - 7y^2 + 9 \\ 2x^2 + 2y^2 - 6 \\ - \quad - \quad + \\ \hline x^2 - 9y^2 + 15 \\ \hline \end{array}$$

[Change of the sign]

**Alternative Method**

$(3x^2 - 7y^2 + 9) - (2x^2 + 2y^2 - 6)$

$= 3x^2 - 7y^2 + 9 - 2x^2 - 2y^2 + 6$

$= (3x^2 - 2x^2) + (-7y^2 - 2y^2) + (9 + 6)$

$= x^2 + (-9y^2) + 15$

$= x^2 - 9y^2 + 15$

(i) $x \times 5y = x \times 5 \times y = 5 \times x \times y = 5xy$

(ii) $2x \times 3y = 2 \times x \times 3 \times y = 2 \times 3 \times x \times y = 6xy$

(iii) $2x \times (-3y) = 2 \times (-3) \times x \times y = -6 \times x \times y = -6xy$

(iv) $2x \times 3x^2 = 2 \times x \times 3 \times x^2 = (2 \times 3) \times (x \times x^2) = 6x^3$

(v) $2x \times (-3xyz) = 2 \times (-3) \times (x \times xyz) = -6x^2yz.$

**Note:** 1. Product of monomials are also monomials.

2. Coefficient of the product = Coefficient of the first monomial × Coefficient of the second monomial.

3. Laws of exponents $a^m \times a^n = a^{m+n}$ is useful, in finding the product of the terms.

4. The products of $a$ and $b$ can be represented as: $a \times b$, $ab$, $a.b$, $a\,(b)$, $(a)\,b$, $(a)\,(b)$, $(ab)$.

(vi) $(3x^2)(4x^3)$

$= (3 \times x \times x)\,(4 \times x \times x \times x)$

$= (3 \times 4)\,(x \times x \times x \times x \times x)$

$= 12x^5$

(Or)

$(3x^2)(4x^3) = (3 \times 4)(x^2 \times x^3) = 12(x^{2+3})$

$= 12x^5$ (using $a^m \times a^n = a^{m+n}$)

Some more useful examples are as follows:

(vii) $2x \times 3y \times 5z = (2x \times 3y) \times 5z$

$= (6xy) \times 5z$

$= 30\ xyz$

(or) $2x \times 3y \times 5z = (2 \times 3 \times 5) \times (x \times y \times z) = 30xyz$

(viii) $4ab \times 3a^2b^2 \times 2a^3b^3 = (4ab \times 3a^2b^2) \times 2a^3b^3$

$= (12a^3b^3) \times 2a^3\ b^3$

$= 24a^6b^6$

(or) $4ab \times 3a^2b^2 \times 2a^3b^3 = 4 \times 3 \times 2 \times (ab \times a^2b^2 \times a^3b^3)$

$= 24(a^{1+2+3} \times b^{1+2+3})$

$= 24a^6b^6$

## 1.3.2 Multiplying a Monomial by a Binomial

Let us learn to multiply a monomial by a binomial through the following examples.

### *Example 1.4*

Simplify: $(2x) \times (3x + 5)$

***Solution*** We can write this as:

Step 1

$(2x) \times (3x + 5) = (2x \times 3x) + (2x \times 5)$ [Using the distributive law]

Step 2

$= 6x^2 + 10x$

### *Example 1.5*

Simplify: $(-2x) \times (4 - 5y)$

***Solution***

Step 1

$(-2x) \times (4 - 5y) = [(-2x) \times 4] + [(-2x) \times (-5y)]$

Step 2

$= (-8x) + (10xy)$ [Using the distributive law]

$= -8x + 10xy$

**Note:** (i) The product of a monomial by a binomial is a binomial.

(ii) We use the commutative and distributive laws to solve multiplication sums. $a \times b = b \times a$ **(Commutative Law)**

$a(b + c) = ab + ac$ and $a(b - c) = ab - ac$ **(Distributive laws)**

### 1.3.3. Multiplying a Monomial by a Polynomial

A Polynomial with more than two terms is multiplied by a monomial as follows:

*Example 1.6*

Simplify: (i) $3(5y^2 - 3y + 2)$

(ii) $2x^2 \times (3x^2 - 5x + 8)$

*Solution*

(i) $3(5y^2 - 3y + 2) = (3 \times 5y^2) + (3 \times -3y) + (3 \times 2)$

$= 15y^2 - 9y + 6$

[or] $\begin{array}{r} 5y^2 - 3y + 2 \\ \times \quad\quad\quad 3 \\ \hline 15y^2 - 9y + 6 \\ \hline \end{array}$

(ii) $2x^2 \times (3x^2 - 5x + 8)$

$= (2x^2 \times 3x^2) + (2x^2 \times (-5x)) + (2x^2 \times 8)$

$= 6x^4 - 10x^3 + 16x^2$

[or] $\begin{array}{r} 3x^2 - 5x + 8 \\ \times \quad\quad 2x^2 \\ \hline 6x^4 - 10x^3 + 16x^2 \\ \hline \end{array}$

## 1.3.4 Multiplying a Binomial by a Binomial

We shall now proceed to multiply a binomial by another binomial, using the distributive and commutative laws. Let us consider the following example.

*Example 1.7*

Simplify : $(2a+3b)(5a+4b)$

***Solution***

**Every term in one binomial multiplies every term in the other binomial.**

$$
\begin{aligned}
(2a+3b)\,(5a+4b) &= (2a\times 5a)+(2a\times 4b)+(3b\times 5a)+(3b\times 4b)\\
&= 10a^2+8ab+15ba+12b^2\\
&= 10a^2+8ab+15ab+12b^2 \qquad [\because ab=ba]\\
&= 10a^2+23ab+12b^2
\end{aligned}
$$

[Adding like terms $8ab$ and $15ab$]

$$(2a+3b)(5a+4b) = 10a^2+23ab+12b^2$$

**Note :** In the above example, while multiplying two binomials we get only 3 terms instead of $2\times 2 = 4$ terms. Because we have combined the like terms $8ab$ and $15ab$.

### 1.3.5 Multiplying a Binomial by a Trinomial

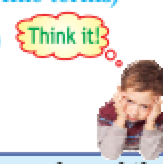
In this multiplication, we have to multiply each of the three terms of the trinomial by each of the two terms in the binomial.

*Example 1.8*

Simplify: $(x+3)(x^2-5x+7)$

*Solution*

$$
\begin{aligned}
(x+3)\,(x^2-5x+7) &= x\,(x^2-5x+7) + 3\,(x^2-5x+7) \quad \text{(Using the distributive law)}\\
&= x^3-5x^2+7x+3x^2-15x+21\\
&= x^3-5x^2+3x^2+7x-15x+21 \quad \text{(Grouping the like terms)}\\
&= x^3-2x^2-8x+21 \quad \text{(Combining the like terms)}
\end{aligned}
$$




Alternative Method :

$$
\begin{array}{lcr}
 & & (x+3)\\
 & & \times\ (x^2-5x+7)\\
x(x^2-5x+7) & : & x^3-5x^2+7x\\
3(x^2-5x+7) & : & 3x^2-15x+21\\
\hline
 & & = x^3-2x^2-8x+21
\end{array}
$$

In this example, while multipltying, instead of expecting $2 \times 3 = 6$ terms, we are getting only 4 terms in the product. **Could you find out the reason?**

### 1.4.1 Algebraic Identities

We proceed now to study the three important Algebraic Identities which are very useful in solving many problems. We obtain these Identities by multiplying a binomial by another binomial.

**Identity 1**

Let us consider $(a+b)^2$.

$$
\begin{aligned}
(a+b)^2 &= (a+b)(a+b)\\
&= a^2+ab+ba+b^2\\
&= a^2+ab+ab+b^2 \qquad [\because ab = ba]\\
&= a^2+2ab+b^2
\end{aligned}
$$

Thus, $\boxed{(a+b)^2 = a^2+2ab+b^2}$

### 1.4.2 Applying the Identities

*Example 1.9*

Expand (i) $(x+5)^2$ (ii) $(x+2y)^2$ (iii) $(2x+3y)^2$ (iv) $105^2$.

*Solution*

(i)

$$\begin{aligned}(x+5)^2 &= x^2+2(x)(5)+5^2\\ &= x^2+10x+25\end{aligned}$$

***Using the identity:***
$(a+b)^2 = a^2+2ab+b^2$
Here, $a = x$, $b = 5$.

Aliter:

$$\begin{aligned}(x+5)^2 &= (x+5)(x+5)\\ &= x(x+5)+5(x+5)\\ &= x^2+5x+5x+25\\ &= x^2+10x+25\end{aligned}$$

(ii)

$$\begin{aligned}(x+2y)^2 &= x^2+2(x)(2y)+(2y)^2\\ &= x^2+4xy+4y^2\end{aligned}$$

***Using the identity:***
$(a+b)^2 = a^2+2ab+b^2$
Here, $a = x$, $b = 2y$.

Aliter:

$$\begin{aligned}(x+2y)^2 &= (x+2y)(x+2y)\\ &= x(x+2y)+2y(x+2y)\\ &= x^2+2xy+2yx+4y^2 && [\because xy = yx]\\ &= x^2+4xy+4y^2\end{aligned}$$

(iii)

$$\begin{aligned}(2x+3y)^2 &= (2x)^2+2(2x)(3y)+(3y)^2\\ &= 4x^2+12xy+9y^2\end{aligned}$$

***Using the identity:***
$(a+b)^2 = a^2+2ab+b^2$
Here, $a = 2x$, $b = 3y$.

Aliter:

$$\begin{aligned}(2x+3y)^2 &= (2x+3y)(2x+3y)\\ &= 2x(2x+3y)+3y(2x+3y)\\ &= (2x)(2x)+(2x)(3y)+(3y)(2x)+(3y)(3y)\\ &= 4x^2+6xy+6yx+9y^2 && [\because xy = yx]\\ (2x+3y)^2 &= 4x^2+12xy+9y^2\end{aligned}$$

(iv)

$$\begin{aligned}105^2 &= (100+5)^2\\ &= 100^2+2(100)(5)+5^2\\ &= (100\times 100)+1000+25\\ &= 10000+1000+25\\ 105^2 &= 11025\end{aligned}$$

***Using the identity:***
$(a+b)^2 = a^2+2ab+b^2$
Here, $a = 100$, $b = 5$.

## Example 1.10

Find the values of (i) $(x-y)^2$ (ii) $(3p-2q)^2$ (iii) $97^2$ (iv) $(4.9)^2$

**Solution**

(i) $(x-y)^2 = x^2 - 2(x)(y) + y^2$

$= x^2 - 2xy + y^2$

*Using the identity:*
$(a-b)^2 = a^2 - 2ab + b^2$
Here, $a = x$, $b = y$.

(ii) $(3p-2q)^2 = (3p)^2 - 2(3p)(2q) + (2q)^2$

$= 9p^2 - 12pq + 4q^2$

*Using the identity:*
$(a-b)^2 = a^2 - 2ab + b^2$
Here, $a = 3p$, $b = 2q$.

(iii) $97^2 = (100-3)^2$

$= (100)^2 - 2(100)(3) + 3^2$

$= 10000 - 600 + 9$

$= 9400 + 9$

$= 9409$

*Using the identity:*
$(a-b)^2 = a^2 - 2ab + b^2$
Here, $a = 100$, $b = 3$.

(iv) $(4.9)^2 = (5.0-0.1)^2$

$= (5.0)^2 - 2(5.0)(0.1) + (0.1)^2$

$= 25.00 - 1.00 + 0.01$

$= 24.01$

*Using the identity:*
$(a-b)^2 = a^2 - 2ab + b^2$
Here, $a = 5.0$, $b = 0.1$.

### Example 1.11

Evaluate the following using the identity $(a+b)(a-b) = a^2 - b^2$

(i) $(x+3)(x-3)$ (ii) $(5a+3b)(5a-3b)$ (iii) $52 \times 48$ (iv) $997^2 - 3^2$.

**Solution**

(i) 
$$(x+3)(x-3) = x^2 - 3^2 = x^2 - 9$$

*Using the identity:* $(a+b)(a-b) = a^2 - b^2$ Here, $a = x$, $b = 3$.

(ii)
$$(5a+3b)(5a-3b) = (5a)^2 - (3b)^2 = 25a^2 - 9b^2$$

*Using the identity:* $(a+b)(a-b) = `a^2 - b^2$ Here, $a = 5a$, $b = 3b$.

(iii)
$$\begin{aligned} 52 \times 48 &= (50+2)(50-2) \\ &= 50^2 - 2^2 \\ &= 2500 - 4 \\ &= 2496 \end{aligned}$$

*Using the identity:* $(a+b)(a-b) = a^2 - b^2$ Here, $a = 50$, $b = 2$.

(iv)
$$\begin{aligned} 997^2 - 3^2 &= (997+3)(997-3) \\ &= (1000)(994) \\ &= 994000 \end{aligned}$$

*Using the identity:* $a^2 - b^2 = (a+b)(a-b)$ Here, $a = 997$, $b = 3$.

## 1.4.3 Deducing some useful Identities

Let us consider,

(i)
$$\begin{aligned} (a+b)^2 + (a-b)^2 &= (a^2 + 2ab + b^2) + (a^2 - 2ab + b^2) \\ &= a^2 + \cancel{2}ab + b^2 + a^2 - \cancel{2}ab + b^2 \\ &= 2a^2 + 2b^2 \\ (a+b)^2 + (a-b)^2 &= 2(a^2 + b^2) \\ \frac{1}{2}[(a+b)^2 + (a-b)^2] &= a^2 + b^2 \end{aligned}$$

(ii)
$$\begin{aligned} (a+b)^2 - (a-b)^2 &= (a^2 + 2ab + b^2) - (a^2 - 2ab + b^2) \\ &= \cancel{a^2} + 2ab + \cancel{b^2} - \cancel{a^2} + 2ab - \cancel{b^2} \\ (a+b)^2 - (a-b)^2 &= 4ab \\ \frac{1}{4}[(a+b)^2 - (a-b)^2] &= ab \end{aligned}$$

(iii) $(a+b)^2 - 2ab = a^2 + b^2 + \cancel{2}ab - \cancel{2}ab$

$= a^2 + b^2$

$$(a+b)^2 - 2ab = a^2 + b^2$$

(iv) $(a+b)^2 - 4ab = a^2 + 2ab + b^2 - 4ab$

$= a^2 - 2ab + b^2$

$= (a-b)^2$

$$(a+b)^2 - 4ab = (a-b)^2$$

(v) $(a-b)^2 + 2ab = a^2 - 2\cancel{a}b + b^2 + 2\cancel{a}b$

$= a^2 + b^2$

$$(a-b)^2 + 2ab = a^2 + b^2$$

(vi) $(a-b)^2 + 4ab = a^2 - 2ab + b^2 + 4ab$

$= a^2 + 2ab + b^2$

$= (a+b)^2$

$$(a-b)^2 + 4ab = (a+b)^2$$

### Deduced Identities

- $\frac{1}{2}[(a+b)^2 + (a-b)^2] = a^2 + b^2$
- $\frac{1}{4}[(a+b)^2 - (a-b)^2] = ab$
- $(a+b)^2 - 2ab = a^2 + b^2$
- $(a+b)^2 - 4ab = (a-b)^2$
- $(a-b)^2 + 2ab = a^2 + b^2$
- $(a-b)^2 + 4ab = (a+b)^2$

If the values of $a+b$ and $a-b$ are 7 and 4 respectively, find the values of $a^2+b^2$ and $ab$.

*Solution*

(i)

$$\begin{aligned} a^2+b^2 &= \frac{1}{2}[(a+b)^2+(a-b)^2] \\ &= \frac{1}{2}[7^2+4^2] \quad \text{[Substituting the values of } a+b=7,\ a-b=4] \\ &= \frac{1}{2}(49+16) \\ &= \frac{1}{2}(65) \\ &= \frac{65}{2} \\ a^2+b^2 &= \frac{65}{2} \end{aligned}$$

(ii)

$$\begin{aligned} ab &= \frac{1}{4}[(a+b)^2-(a-b)^2] \\ &= \frac{1}{4}(7^2-4^2) \quad \text{[Substituting the values of } a+b=7,\ a-b=4] \\ &= \frac{1}{4}(49-16) \\ &= \frac{1}{4}(33) \\ ab &= \frac{33}{4} \end{aligned}$$

*Example 1.14*

If $(a+b)=10$ and $ab=20$, find $a^2+b^2$ and $(a-b)^2$.

*Solution*

(i)

$$\begin{aligned} a^2+b^2 &= (a+b)^2-2ab \quad \text{[Substituting } a+b=10,\ ab=20] \\ a^2+b^2 &= (10)^2-2(20) \\ &= 100-40=60 \\ a^2+b^2 &= 60 \end{aligned}$$

(ii)

$$\begin{aligned} (a-b)^2 &= (a+b)^2-4ab \quad \text{[Substituting } a+b=10,\ ab=20] \\ &= (10)^2-4(20) \\ &= 100-80 \\ (a-b)^2 &= 20 \end{aligned}$$

### Example 1.15

If $(x+l)(x+m) = x^2 + 4x + 2$ find $l^2 + m^2$ and $(l-m)^2$

**Solution**

By product formula, we know

$$(x+l)(x+m) = x^2 + (l+m)x + lm$$

So, by comparing RHS with $x^2 + 4x + 2$, we have,

$$l + m = 4 \text{ and } lm = 2$$

Now,

$$\begin{aligned} l^2 + m^2 &= (l+m)^2 - 2lm \\ &= 4^2 - 2(2) = 16 - 4 \\ l^2 + m^2 &= 12 \\ (l-m)^2 &= (l+m)^2 - 4lm \\ &= 4^2 - 4(2) = 16 - 8 \\ (l-m)^2 &= 8 \end{aligned}$$

### Example 1.21

Factorize: $x^2 + 6x + 8$

**Solution**

Comparing $x^2 + 6x + 8$ with $x^2 + (a+b)x + ab = (x+a)(x+b)$,

we get $ab = 8$ and $a + b = 6$.

$$\begin{aligned} \therefore\ x^2 + 6x + 8 &= x^2 + (2+4)x + (2 \times 4) \\ &= (x+2)(x+4) \end{aligned}$$

The factors of $x^2 + 6x + 8$ are $(x+2)$ and $(x+4)$.

| Factors of 8 | Sum of factors |
|---|---|
| 1, 8 | 9 |
| 2, 4 | 6 |

Hence the correct factors are 2, 4

Solve: $(5x^2 + 10x) \div (x + 2)$.

**Solution**

$$(5x^2 + 10x) \div (x + 2) = \frac{5x^2 + 10x}{x + 2}$$

Let us factorize the numerator $(5x^2 + 10x)$.

$$5x^2 + 10x = (5 \times x \times x) + (5 \times 2 \times x)$$

$$= 5x(x + 2) \quad \text{[Taking out the common factor } 5x\text{]}$$

Now, $$(5x^2 + 10x) \div (x + 2) = \frac{5x^2 + 10x}{x + 2}$$

$$= \frac{5x\cancel{(x+2)}}{\cancel{(x+2)}} = 5x. \quad \text{[By cancelling } (x+2)\text{]}$$

*Example 1.26*

Solve: $2x + 5 = 23 - x$

**Solution**

$$2x + 5 = 23 - x$$

$$2x + 5 - 5 = 23 - x - 5$$

[Adding – 5 both sides]

$$2x = 18 - x$$

$$2x + x = 18 - x + x$$

$$3x = 18$$

$$\frac{3x}{3} = \frac{18}{3} \quad \text{[Dividing both the sides by 3]}$$

$$x = 6$$

**Alternative Method**

$$2x + 5 = 23 - x$$

$$2x + x = 23 - 5 \quad \text{[By transposition]}$$

$$3x = 18$$

$$x = \frac{18}{3} \quad \text{[Dividing both sides by 3]}$$

$$x = 6$$

Verification: LHS $= 2x + 5 = 2\,(6) + 5 = 17$,

RHS $= 23 - x = 23 - 6 = 17$.

## Example 1.27

Solve : $\frac{9}{2}m + m = 22$

**Solution**

$$\frac{9}{2}m + m = 22$$

$$\frac{9m + 2m}{2} = 22 \quad \text{[Taking LCM on LHS]}$$

$$\frac{11m}{2} = 22$$

$$m = \frac{22 \times 2}{11} \quad \text{[By cross multiplication]}$$

$$m = 4$$

**Verification:**

$$\text{LHS} = \frac{9}{2}m + m = \frac{9}{2}(4) + 4$$

$$= 18 + 4 = 22 = \text{RHS}$$

## Example 1.28

Solve: $\frac{2}{x} - \frac{5}{3x} = \frac{1}{9}$

**Solution**

$$\frac{2}{x} - \frac{5}{3x} = \frac{1}{9}$$

$$\frac{6-5}{3x} = \frac{1}{9} \quad \text{[Taking LCM on LHS]}$$

$$\frac{1}{3x} = \frac{1}{9}$$

$$3x = 9;\ x = \frac{9}{3};\ x = 3.$$

**Verification:**

$$\text{LHS} = \frac{2}{x} - \frac{5}{3x}$$

$$= \frac{2}{3} - \frac{5}{3(3)} = \frac{2}{3} - \frac{5}{9}$$

$$= \frac{6-5}{9} = \frac{1}{9} = \text{RHS}$$

## Example 1.29

Find the two consecutive positive odd integers whose sum is 32.

**Solution**

Let the two consecutive positive odd integers be $x$ and $(x + 2)$.

Then, their sum is 32.

$$\therefore (x) + (x + 2) = 32$$

$$2x + 2 = 32$$

$$2x = 32 - 2$$

$$2x = 30$$

$$x = \frac{30}{2} = 15$$

Since $x = 15$, then the other integer, $x + 2 = 15 + 2 = 17$

$\therefore$ The two required consecutive positive odd integers are 15 and 17.

**Verification:**

$15 + 17 = 32$

*Example 1.30*

One third of one half of one fifth of a number is 15. Find the number.

*Solution*

Let the required number be $x$.

$$\text{Then, } \frac{1}{3} \text{ of } \frac{1}{2} \text{ of } \frac{1}{5} \text{ of } x = 15.$$

$$\text{i.e. } \frac{1}{3}\times\frac{1}{2}\times\frac{1}{5}\times x = 15$$

$$x = 15\times3\times2\times5$$

$$x = 45 \times 10 = 450$$

Hence the required number is 450.

**Verification:**

$$\text{LHS} = \frac{1}{3}\times\frac{1}{2}\times\frac{1}{5}\times x$$

$$= \frac{1}{3}\times\frac{1}{2}\times\frac{1}{5}\times 450$$

$$= 15 = \text{RHS}$$

*Example 1.31*

A rational number is such that when we multiply it by $\frac{5}{2}$ and add $\frac{2}{3}$ to the product we get $\frac{-7}{12}$. What is the number?

*Solution*

Let the rational number be $x$.

When we multiply it by $\frac{5}{2}$ and add $\frac{2}{3}$ to the product we get $\frac{-7}{12}$.

$$\text{i.e., } x\times\frac{5}{2}+\frac{2}{3} = \frac{-7}{12}$$

$$\frac{5x}{2} = \frac{-7}{12} - \frac{2}{3}$$

$$= \frac{-7-8}{12}$$

$$= \frac{-15}{12}$$

$$x = \frac{-15}{12}\times\frac{2}{5}$$

$$= \frac{-1}{2}.$$

Hence the required number is $\frac{-1}{2}$.

**Verification:**

$$\text{LHS} = \frac{-1}{2}\times\frac{5}{2}+\frac{2}{3} = \frac{-5}{4}+\frac{2}{3}$$

$$= \frac{-15+8}{12} = \frac{-7}{12} = \text{RHS.}$$

### Example 1.32

Arun is now half as old as his father. Twelve years ago the father's age was three times as old as Arun. Find their present ages.

**Solution**

Let Arun's age be $x$ years now.

Then his father's age $= 2x$ years

12 years ago, Arun's age was $(x - 12)$ years and

his faher's age was $(2x - 12)$ years.

Given that,

$$(2x - 12) = 3(x - 12)$$
$$2x - 12 = 3x - 36$$
$$36 - 12 = 3x - 2x$$
$$x = 24$$

Therefore, Arun's present age $= 24$ years.

His father's present age $= 2(24) = 48$ years.

**Verification:**

| Arun's age | Father's age |
|---|---|
| Now : 24 | 48 |
| 12 years ago<br>24 – 12 = 12 | 48 – 12 = 36<br>36 = 3 (Arun's age)<br>= 3 (12) = 36 |

### Example 1.33

By selling a car for ₹ 1,40,000, a man suffered a loss of 20%. What was the cost price of the car?

**Solution**

Let the cost price of the car be $x$.

Loss of 20% $= \frac{20}{100}$ of $x = \frac{1}{5} \times x = \frac{x}{5}$

We know that,

$$\text{Cost price} - \text{Loss} = \text{Selling price}$$
$$x - \frac{x}{5} = 140000$$
$$\frac{5x - x}{5} = 140000$$
$$\frac{4x}{5} = 140000$$
$$x = 140000 \times \frac{5}{4}$$
$$x = 175000$$

Hence the cost price of the car is ₹ 1,75,000.

**Verification:**

Loss $=$ 20% of 175000

$= \frac{20}{100} \times 175000$

$=$ ₹ 35,000

S.P $=$ C.P - Loss

$= 175000 - 35000$

$= 140000$

**RESULTS ON PROFIT, LOSS AND SIMPLE INTEREST**

(i) Profit or Gain = Selling price – Cost price

(ii) Loss = Cost price – Selling price

(iii) Profit % = $\frac{\text{Profit}}{\text{C.P.}} \times 100.$

(iv) Loss % = $\frac{\text{Loss}}{\text{C.P.}} \times 100$

(v) Simple interest (I) = $\frac{\text{Principal} \times \text{Time} \times \text{Rate}}{100} = \frac{Pnr}{100}$

(vi) Amount = Principal + Interest

## 1.3.1. Application of Percentage

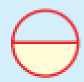
We have already learnt percentages in the previous classes. We present these ideas as follows:

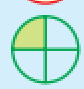
(i) Two percent = $2\% = \frac{2}{100}$

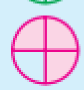
(ii) 8% of 600 kg = $\frac{8}{100} \times 600 = 48$ kg

(ii) 125% = $\frac{125}{100} = \frac{5}{4} = 1\frac{1}{4}$

Do you know?

 $= \frac{1}{2} = 50\ \%$

 $= \frac{1}{4} = 25\ \%$

 $= \frac{3}{4} = 75\ \%$

Now, we learn to apply percentages in some problems.

*Example 1.1*

What percent is 15 paise of 2 rupees 70 paise?

***Solution***

2 rupees 70 paise = $(2 \times 100$ paise $+ 70$ paise$)$

= 200 paise + 70 paise

= 270 paise

Required percentage = $\frac{15}{270} \times 100 = \frac{50}{9} = 5\frac{5}{9}\%$.

### Example 1.2

Find the total amount if 12% of it is ₹ 1080.

**Solution**

Let the total amount be $x$ .

**Given :** 12% of the total amount = ₹ 1080

$$\frac{12}{100} \times x = 1080$$

$$x = \frac{1080 \times 100}{12} = ₹\ 9000$$

$\therefore$ The total amount = ₹ 9000.

### Example 1.3

72% of 25 students are good in Mathematics. How many are not good in Mathematics?

**Solution**

Percentage of students good in Mathematics = 72%

Number of students good in Mathematics = 72% of 25 students

$$= \frac{72}{100} \times 25 = 18 \text{ students}$$

Number of students not good in Mathematics = 25 – 18 = 7.

### Example 1.4

Find the number which is 15% less than 240.

**Solution**

$$15\% \text{ of } 240 = \frac{15}{100} \times 240 = 36$$

$\therefore$ The required number = 240 – 36 = 204.

### Example 1.5

The price of a house is decreased from Rupees Fifteen lakhs to Rupees Twelve lakhs. Find the percentage of decrease.

**Solution**

Original price = ₹ 15,00,000

Change in price = ₹ 12,00,000

Decrease in price = 15,00,000 – 12,00,000 = 3,00,000

$$\therefore \text{Percentage of decrease} = \frac{300000}{1500000} \times 100 = 20\%$$

**Remember**

$$\text{Percentage of increase} = \frac{\text{Increase in amount}}{\text{Original amount}} \times 100$$

$$\text{Percentage of decrease} = \frac{\text{Decrease in amount}}{\text{Original amount}} \times 100$$

### (i) Illustration of the formula for S.P.

Consider the following situation:


Rajesh buys a pen for ₹ 80 and sells it to his friend. If he wants to make a profit of 5%, can you say the price for which he would have sold?

Rajesh bought the pen for ₹ 80 which is the Cost Price (C.P.). When he sold, he makes a profit of 5% which is calculated on the C.P.

$$\therefore \text{Profit} = 5\% \text{ of C.P.} = \frac{5}{100} \times 80 = ₹\, 4$$

Since there is a profit, S.P. > C.P.

$$\textbf{S.P.} = \textbf{C.P.} + \textbf{Profit}$$
$$= 80 + 4 = ₹\, 84.$$

$\therefore$ The price for which Rajesh would have sold = ₹ 84.

The same problem can be done using the formula.

$$\text{Selling price (S.P.)} = \frac{\mathbf{(100 + Profit\%)}}{\mathbf{100}} \times \mathbf{C.P.}$$

$$= \frac{(100+5)}{100} \times 80 = \frac{105}{100} \times 80 = ₹\, 84.$$

## (ii) Illustration of the formula for C.P.

Consider the following situation:

Suppose a shopkeeper sells a wrist watch for ₹ 540 making a profit of 5 %, then what would have been the cost of the watch?

The shopkeeper had sold the watch at a profit of 5 % on the C.P. Since C.P. is not known, let us take it as ₹ 100.

Profit of 5% is made on the C.P.

$$\therefore \text{Profit} = 5\% \text{ of C.P.} = \frac{5}{100} \times 100 = ₹\,5.$$

We know,

$$\textbf{S.P.} = \textbf{C.P.} + \textbf{Profit} = 100 + 5 = ₹\,105.$$

Here , if S.P. is ₹ 105, then C.P. is ₹ 100.

When S.P. of the watch is ₹ 540, C.P. $= \frac{540 \times 100}{105} = ₹\,514.29$

∴ The watch would have cost ₹ 514.29 for the shopkeeper.

The above problem can also be solved by using the formula method.

$$\textbf{C.P.} = \left(\frac{\textbf{100}}{\textbf{100 + profit \%}}\right) \times \textbf{S.P.}$$

$$= \frac{100}{100+5} \times 540$$

$$= \frac{100}{105} \times 540$$

$$= ₹\,514.29.$$

We now summarize the formulae to calculate S.P. and C.P. as follows:

| | |
|---|---|
| 1. When there is a profit<br>(i) C.P. $= \left(\frac{100}{100 + \text{profit}\%}\right) \times$ S.P. | 1. When there is a loss<br>(ii) C.P. $= \left(\frac{100}{100 - \text{loss}\%}\right) \times$ S.P. |
| 2. When there is a profit<br>(i) S.P. $= \left(\frac{100 + \text{profit}\,\%}{100}\right) \times$ C.P. | 2. When there is a loss,<br>(ii) S.P. $= \left(\frac{100 - \text{loss}\%}{100}\right) \times$ C.P. |

***Example 1.6***

Hameed buys a colour T.V set for ₹ 15,200 and sells it at a loss of 20%. What is the selling price of the T.V set?

***Solution***

Raghul used this method:

Loss is 20% of the C.P.

$$= \frac{20}{100} \times 15200$$

$$= ₹\ 3040$$

$$\text{S.P.} = \text{C.P.} - \text{Loss}$$

$$= 15{,}200 - 3{,}040$$

$$= ₹\ 12{,}160$$

OR

Roshan used the formula method:

$$\text{C.P.} = ₹\ 15,200$$

$$\text{Loss} = 20\%$$

$$\text{S.P.} = \frac{100 - \text{Loss}\%}{100} \times \text{C.P.}$$

$$= \frac{100 - 20}{100} \times 15200$$

$$= \frac{80}{100} \times 15200$$

$$= ₹\ 12{,}160$$

Both Raghul and Roshan came out with the same answer that the selling price of the T.V. set is ₹ 12,160.

### *Example 1.7*

A scooty is sold for ₹ 13,600 and fetches a loss of 15%. Find the cost price of the scooty.

Devi used this method:

Loss of 15% means,

if C.P. is ₹ 100, Loss = ₹ 15

Therefore, S.P. would be

$(100 - 15) = ₹\ 85$

If S.P. is ₹ 85, C.P. is ₹ 100

When S.P. is ₹13,600, then

$$\text{C.P.} = \frac{100 \times 13600}{85} = ₹\ 16,000$$

OR

Revathi used the formula method:

$$\text{Loss} = 15\%.$$

$$\text{S.P.} = ₹\ 13,600$$

$$\text{C.P.} = \frac{100}{100 - \text{Loss}\%} \times \text{S.P.} = \frac{100}{100 - 15} \times 13600 = \frac{100}{85} \times 13600 = ₹\ 16,000$$

Hence the cost price of the scooty is ₹ 16,000.

### *Example 1.8*

The cost price of 11 pens is equal to the selling price of 10 pens. Find the loss or gain percent.

***Solution***

Let S.P. of each pen be $x$.

$$\text{S.P. of 10 pens} = ₹\ 10x$$

$$\text{S.P. of 11 pens} = ₹\ 11x$$

**Given** : C.P. of 11 pens = S.P. of 10 pens = ₹ $10x$

Here, S.P. > C.P.

$$\therefore \text{Profit} = \text{S.P.} - \text{C.P.} = 11x - 10x = x$$

$$\text{Profit}\ \% = \frac{\text{Profit}}{\text{C.P.}} \times 100 = \frac{x}{10x} \times 100 = 10\%.$$

### *Example 1.9*

A man sells two wrist watches at ₹ 594 each. On one he gains 10% and on the other he loses 10%. Find his gain or loss percent on the whole.

*Solution*

**Given :** S.P. of the first wrist watch = ₹ 594, Gain% = 10%

$$\therefore \text{ C.P. of the first wrist watch } = \frac{100}{100+\text{profit\%}} \times \text{S.P.}$$

$$= \frac{100}{(100+10)} \times 594$$

$$= \frac{100}{110} \times 594 = ₹\ 540.$$

Similarly, C.P. of the second watch on which he loses 10% is

$$= \frac{100}{(100-\text{Loss\%})} \times \text{S.P.}$$

$$= \frac{100}{(100-10)} \times 594 = \frac{100}{90} \times 594 = ₹\ 660.$$

To say whether there was an overall Profit or Loss, we need to find the combined C.P. and S.P.

Total C.P. of the two watches = 540 + 660 = ₹ 1,200.

Total S.P. of the two watches = 594 + 594 = ₹ 1,188.

Net Loss = 1,200 – 1,188 = ₹ 12.

$$\text{Loss\%} = \frac{\text{Loss}}{\text{C.P.}} \times 100$$

$$= \frac{12}{1200} \times 100 = 1\%.$$

### *Example 1.10*

Raju bought a motorcycle for ₹ 36,000 and then bought some extra fittings to make it perfect and good looking. He sold the bike at a profit of 10% and he got ₹ 44,000. How much did he spend to buy the extra fittings made for the motorcycle?

*Solution*

Let the C.P. be ₹ 100.

$$\text{Profit} = 10\%, \text{ S.P.} = ₹\ 110$$

If S.P. is ₹ 110, then C.P. is ₹ 100.

When S.P. is ₹ 44,000

$$\text{C.P.} = \frac{44000 \times 100}{110} = ₹\ 40{,}000$$

$\therefore$ Amount spent on extra fittings = 40,000 – 36,000 = ₹ 4,000.

### 1.3.3. Application of Overhead Expenses

Maya went with her father to purchase an Air cooler. They bought it for ₹ 18,000. The shop wherein they bought was not closer to their residence. So they had to arrange a vehicle to take the air cooler to their residence. They paid conveyance charges of ₹ 500. Hence the C.P. of the air cooler is not only ₹ 18,000 but it also includes the **Conveyance Charges** (Transportation charges) ₹ 500 which is called as **Overhead Expenses .**

Now,

$$\begin{aligned} \text{C.P. of the air cooler} &= \text{Real cost} + \text{Conveyance charges} \\ &= 18{,}000 + 500 = ₹\ 18{,}500 \end{aligned}$$

Consider another situation, where Kishore's father buys an old Maruti car from a Chennai dealer for ₹ 2,75,000 and spends ₹ 25,000 for painting the car. And then he transports the car to his native village for which he spends again ₹ 2,000. Can you answer the following questions:

(i) What is the the overall cost price of the car?

(ii) What is the real cost price of the car?

(iii) What are the overhead expenses referred here?

In the above example the painting charges and the transportation charges are the overhead expenses.

$$\begin{aligned}\therefore \text{Cost price of the car} &= \text{Real cost price} + \text{Overhead expenses}\\ &= 2{,}75{,}000 + (25{,}000 + 2{,}000)\\ &= 2{,}75{,}000 + 27{,}000 = ₹\ 3{,}02{,}000.\end{aligned}$$

Sometimes when an article is bought or sold, some additional expenses occur while buying or before selling it. These expenses have to be included in the cost price. These expenses are referred to as **Overhead Expenses**. These may include expenses like amount spent on repairs, labour charges, transportation, etc.,

$$\begin{aligned}\text{Discount} &= \text{Marked Price} - \text{Selling Price}\\ \text{Selling Price} &= \text{Marked Price} - \text{Discount}\\ \text{Marked Price} &= \text{Selling Price} + \text{Discount}\end{aligned}$$

### *Example 1.11*

A bicycle marked at ₹ 1,500 is sold for ₹ 1,350. What is the percentage of discount?

#### *Solution*

**Given :** Marked Price = ₹ 1500, Selling Price = ₹ 1350

Amount of discount = Marked Price – Selling Price

= 1500 – 1350

= ₹ 150

Discount for ₹ 1500 = ₹ 150

Discount for ₹ 100 = $\frac{150}{1500} \times 100$

Percentage of discount = 10%.

Since Discount is on Marked Price, we will have calculate discount on M.P.

### *Example 1.12*

The list price of a frock is ₹ 220. A discount of 20% on sales is announced. What is the amount of discount on it and its selling price?

#### *Solution*

**Given :** List (Marked) Price of the frock = ₹ 220, Rate of discount = 20%

Amount of discount = $\frac{20}{100} \times 220$

= ₹ 44

∴ Selling Price of the frock = Marked Price – Discount

= 220 – 44

= ₹ 176.

*Example 1.13*

An almirah is sold at ₹ 5,225 after allowing a discount of 5%. Find its marked price.

*Solution*

Krishna used this method:

The discount is given in percentage.

Hence, the M.P. is taken as ₹ 100.

Rate of discount = 5%

Amount of discount $= \frac{5}{100} \times 100$

= ₹ 5.

Selling Price = M.P. – Discount

= 100 – 5 = ₹ 95

If S.P. is ₹ 95, then M.P. is ₹ 100.

When S.P. is ₹ 5225,

M.P. $= \frac{100}{95} \times 5225$

∴ M.P. of the almirah = ₹ 5,500.

[OR]

Vignesh used the formula method:

S.P. = Rs 5225

Discount = 5%

M.P. = ?

$$\text{M.P.} = \left(\frac{100}{100 - \text{Discount\%}}\right) \times \text{S.P.}$$

$$= \left(\frac{100}{100 - 5}\right) \times 5225$$

$$= \frac{100}{95} \times 5225$$

= ₹ 5,500.

### *Example 1.14*

A shopkeeper allows a discount of 10% to his customers and still gains 20%. Find the marked price of an article which costs ₹ 450 to the shopkeeper.

**Solution**

Vanitha used this method:

Let M.P. be ₹ 100.

Discount = 10% of M. P.

$= \frac{10}{100}$ of M.P. $= \frac{10}{100} \times 100$

= ₹ 10

S.P. = M.P. – Discount

= 100 – 10 = ₹ 90

Gain = 20% of C.P.

$= \frac{20}{100} \times 450 =$ ₹ 90

S.P. = C.P. + Gain

= 450 + 90 = ₹ 540.

If S.P. is ₹ 90, then M.P. is ₹ 100.

When S.P. is ₹ 540,

M.P. $= \frac{540 \times 100}{90} =$ ₹ 600

∴ The M.P. of an article = ₹ 600

[OR]

Vimal used the formula method:

Discount = 10%, Gain = 20%,

C.P. = ₹ 450, M.P. = ?

$$\text{M.P.} = \frac{100 + \text{Gain\%}}{100 - \text{Discount\%}} \times \text{C.P.}$$

$$= \frac{(100 + 20)}{(100 - 10)} \times 450$$

$$= \frac{120}{90} \times 450$$

= ₹ 600

*Example 1.15*

A dealer allows a discount of 10% and still gains 10%. What is the cost price of the book which is marked at ₹ 220?

**Solution**

Sugandan used this method:

M.P. = ₹ 220.

Discount = 10% of M.P.

$= \frac{10}{100} \times 220 = ₹\ 22$

S.P. = M.P. – Discount

= 220 – 22 = ₹ 198

Let C.P. be ₹ 100.

Gain = 10% of C. P.

$= \frac{10}{100} \times 100 = ₹\ 10$

S.P. = C.P. + Gain

= 100 + 10

= ₹ 110.

If S.P. is ₹ 110, then C.P. is ₹ 100.

When S.P. is ₹ 198,

C.P. $= \frac{198 \times 100}{110}$

= ₹ 180.

[OR]

Mukundan used the formula method:

Discount = 10%

Gain = 10%

M.P. = ₹ 220

C.P. $= \frac{100 - \text{Discount\%}}{100 + \text{Gain\%}} \times \text{M.P.}$

$= \frac{100 - 10}{100 + 10} \times 220$

$= \frac{90}{110} \times 220 = ₹\ 180.$

*Example 1.16*

A television set was sold for ₹ 14,400 after giving successive discounts of 10% and 20% respectively. What was the marked price?

*Solution*

Selling Price = ₹ 14,400

Let the M.P. be ₹ 100.

First discount = 10% = $\frac{10}{100} \times 100$ = ₹ 10

S.P. after the first discount = 100 – 10 = ₹ 90

Second discount = 20% = $\frac{20}{100} \times 90$ = ₹ 18

Selling Price after the second discount = 90 – 18 = ₹ 72

If S.P. is ₹ 72, then M.P. is ₹ 100.

When S.P. is ₹ 14,400,

M.P. = $\frac{14400 \times 100}{72}$ = ₹ 20,000

M.P. = ₹ 20,000.

*Example 1.17*

A trader buys an article for ₹ 1,200 and marks it 30% above the C.P. He then sells it after allowing a discount of 20%. Find the S.P. and profit percent.

*Solution:*

Let C.P. of the article be ₹ 100

M.P. = 30% above C.P. = ₹ 130

If C.P. is ₹ 100, then M.P. is ₹ 130.

When C.P. is ₹ 1200, M.P. = $\frac{1200 \times 130}{100}$ = ₹ 1560

Discount = 20% of 1560 = $\frac{20}{100} \times 1560$ = ₹ 312

S.P. = M.P. – Discount

= 1560 – 312 = ₹ 1248

Profit = S.P. – C.P.

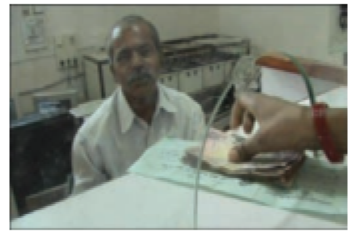
= 1248 –1200 = ₹ 48.

∴ Profit % = $\frac{\text{Profit}}{\text{C.P.}} \times 100$

= $\frac{48}{1200} \times 100 = 4\%$

## 1.4. Compound Interest

In class VII, we have learnt about Simple Interest and the formula for calculating Simple Interest and Amount. In this chapter, we shall discuss the concept of Compound Interest and the method of calculating Compound Interest and Amount at the end of a certain specified period.



Vinay borrowed ₹ 50,000 from a bank for a fixed time period of 2 years. at the rate of 4% per annum.

Vinay has to pay for the first year,

$$\text{Simple interest} = \frac{P \times n \times r}{100}$$

$$= \frac{50000 \times 1 \times 4}{100} = ₹\ 2{,}000$$

Suppose he fails to pay the simple interest ₹ 2,000 at the end of first year, then the interest ₹ 2,000 is added to the old Principal ₹ 50,000 and now the sum =

P + I = ₹ 52,000 becomes the new Principal for the second year for which the interest is calculated.

Now in the second year he will have to pay an interest of

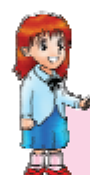
$$\text{S.I.} = \frac{P \times n \times r}{100}$$

$$= \frac{52000 \times 1 \times 4}{100} = ₹\ 2{,}080$$

Therefore Vinay will have to pay more interest for the second year.

This way of calculating interest is called **Compound Interest.**

Generally in banks, insurance companies, post offices and in other companies which lend money and accept deposits, compound interest is followed to find the interest.



*Do you know?*

When the interest is paid on the Principal only, it is called **Simple Interest.** But if the interest is paid on the Principal as well as on the accrued interest, it is called **Compound Interest.**

*Example 1.21*

Ramlal deposited ₹ 8,000 with a finance company for 3 years at an interest of 15% per annum. What is the compound interest that Ramlal gets after 3 years?

*Solution*

**Step 1**: Principal for the first year = ₹ 8,000

Interest for the first year $= \frac{P \times n \times r}{100}$

$= \frac{8000 \times 1 \times 15}{100} =$ ₹ 1,200

Amount at the end of **first year** = P + I = 8,000 + 1,200 = ₹ 9,200

**Step 2**: The amount at the end of the first year becomes the Principal for the second year.

Principal for the second year = ₹ 9,200

Interest for the second year $= \frac{P \times n \times r}{100}$

$= \frac{9200 \times 1 \times 15}{100} =$ ₹ 1,380

Amount at the end of **second year** = P + I = 9,200 + 1,380 = ₹ 10,580

**Step 3**: The amount at the end of the second year becomes the Principal for the third year.

Principal for the third year = ₹ 10,580

Interest for the third year $= \frac{P \times n \times r}{100}$

$= \frac{10580 \times 1 \times 15}{100} =$ ₹ 1,587

Amount at the end of **third year** = P + I

= 10,580 + 1,587 = ₹ 12,167

Hence, the Compound Interest that Ramlal gets after three years is

A – P = 12,167 – 8,000 = ₹ 4,167.

## Deduction of formula for Compound Interest

The above method which we have used for the calculation of Compound Interest is quite lengthy and cumbersome, especially when the period of time is very large. Hence we shall obtain a formula for the computation of Amount and Compound Interest.

If the Principal is P, Rate of interest per annum is $r$ % and the period of time or the number of years is $n$, then we deduce the compound interest formula as follows:

**Step 1 :** Principal for the first year $= \mathrm{P}$

Interest for the first year $= \dfrac{\mathrm{P} \times n \times r}{100}$

$$= \frac{\mathrm{P} \times 1 \times r}{100} = \frac{\mathrm{P}r}{100}$$

Amount at the end of **first year** $= \mathrm{P} + \mathrm{I}$

$$= \mathrm{P} + \frac{\mathrm{P}r}{100}$$

$$= \mathrm{P}\left(1 + \frac{r}{100}\right)$$

**Step 2 :** Principal for the second year $= \mathrm{P}\left(1 + \dfrac{r}{100}\right)$

Interest for the second year $= \dfrac{\mathrm{P}\left(1 + \frac{r}{100}\right) \times 1 \times r}{100}$

(using the S.I.formula)

$$= \mathrm{P}\left(1 + \frac{r}{100}\right) \times \frac{r}{100}$$

Amount at the end of **second year** $= \mathrm{P} + \mathrm{I}$

$$= \mathrm{P}\left(1 + \frac{r}{100}\right) + \mathrm{P}\left(1 + \frac{r}{100}\right) \times \frac{r}{100}$$

$$= \mathrm{P}\left(1 + \frac{r}{100}\right)\left(1 + \frac{r}{100}\right)$$

$$= \mathrm{P}\left(1 + \frac{r}{100}\right)^2$$

**Step 3 :** Principal for the third year $= P\left(1+\frac{r}{100}\right)^2$

Interest for the third year $= \frac{P\left(1+\frac{r}{100}\right)^2 \times 1 \times r}{100}$

(using the S.I.formula)

$$= P\left(1+\frac{r}{100}\right)^2 \times \frac{r}{100}$$

Amount at the end of **third year** $= P + I$

$$= P\left(1+\frac{r}{100}\right)^2 + P\left(1+\frac{r}{100}\right)^2 \times \frac{r}{100}$$

$$= P\left(1+\frac{r}{100}\right)^2 \left(1+\frac{r}{100}\right)$$

$$= \mathbf{P\left(1+\frac{r}{100}\right)^3}$$

Similarly, Amount at the end of $n^{th}$ year is $A = P\left(1+\frac{r}{100}\right)^n$

and C. I. at the end of '$n$' years is given by $A - P$

(i. e.) $C.\ I. = P\left(1+\frac{r}{100}\right)^n - P$

## To Compute Compound Interest

### Case 1: Compounded Annually

When the interest is added to the Principal at the end of each year, we say that the interest is compounded annually.

Here $A = P\left(1+\frac{r}{100}\right)^n$ and C.I. $= A - P$

### Case 2: Compounded Half - Yearly (Semi - Annually)

When the interest is compounded Half - Yearly, there are two conversion periods in a year each after 6 months. In such situations, the Half - Yearly rate will be half of the annual rate, that is $\left(\frac{r}{2}\right)$.

In this case, $A = P\left[1 + \frac{1}{2}\left(\frac{r}{100}\right)\right]^{2n}$ and C.I. = A – P

### Case 3: Compounded Quarterly

When the interest is compounded quarterly, there are four conversion periods in a year and the quarterly rate will be one-fourth of the annual rate, that is $\left(\frac{r}{4}\right)$.

In this case, $A = P\left[1 + \frac{1}{4}\left(\frac{r}{100}\right)\right]^{4n}$ and C.I. = A – P

### Case 4: Compounded when time being fraction of a year

**When interest is compounded annually but time being a fraction.**

In this case, when interest is compounded annually but time being a fraction of a year, say $5\frac{1}{4}$ years, then amount A is given by

$$A = \underbrace{P\left(1 + \frac{r}{100}\right)^5}_{\text{for 5 years}} \underbrace{\left[1 + \frac{1}{4}\left(\frac{r}{100}\right)\right]}_{\text{for } \frac{1}{4} \text{ of year}} \text{ and C.I.} = A - P$$

## Example 1.22

Find the C.I. on ₹ 15,625 at 8% p.a. for 3 years compounded annually.

**Solution**

We know,

$$\text{Amount after 3 years} = P\left(1+\frac{r}{100}\right)^3$$
$$= 15625\left(1+\frac{8}{100}\right)^3$$
$$= 15625\left(1+\frac{2}{25}\right)^3$$
$$= 15625\left(\frac{27}{25}\right)^3$$
$$= 15625\times\frac{27}{25}\times\frac{27}{25}\times\frac{27}{25}$$
$$= ₹\ 19{,}683$$

$$\text{Now, Compound interest} = A - P = 19{,}683 - 15{,}625$$
$$= ₹\ 4{,}058$$

### To find the C.I. when the interest is compounded annually or half-yearly

Let us see what happens to ₹ 100 over a period of one year if an interest is compounded annually or half-yearly.

| S.No | Annually | Half-yearly |
|---|---|---|
| 1 | P = ₹ 100 at 10% per annum compounded annually | P = ₹ 100 at 10% per annum compounded half-yearly |
| 2 | The time period taken is 1 year | The time period is 6 months or ½ year. |
| 3 | $I = \frac{100\times10\times1}{100} = ₹\ 10$ | $I = \frac{100\times10\times1/2}{100} = ₹\ 5$ |
| 4 | A = 100 + 10 = ₹ 110 | A = 100 + 5 = ₹ 105<br>For the next 6 months, P = ₹ 105<br>So, $I = \frac{105\times10\times1/2}{100} = ₹\ 5.25$<br>and A = 105 + 5.25 = ₹ 110.25 |
| 5 | A = ₹ 110 | A = ₹ 110.25 |

### *Example 1.23*

Find the compound interest on ₹ 1000 at the rate of 10% per annum for 18 months when interest is compounded half-yearly.

### *Solution*

Here, P = ₹ 1000, $r$ = 10% per annum

and $n = 18 \text{ months} = \frac{18}{12} \text{ years} = \frac{3}{2} \text{ years} = 1\frac{1}{2} \text{ years}$

$$\therefore \text{Amount after 18 months} = P\left[1 + \frac{1}{2}\left(\frac{r}{100}\right)\right]^{2n}$$

$$= 1000\left[1 + \frac{1}{2}\left(\frac{10}{100}\right)\right]^{2\times\frac{3}{2}}$$

$$= 1000\left(1 + \frac{10}{200}\right)^3$$

$$= 1000\left(\frac{21}{20}\right)^3$$

$$= 1000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}$$

$$= ₹\ 1157.625$$

$$= ₹\ 1157.63$$

$$\text{C. I.} = A - P$$

$$= 1157.63 - 1000 = ₹\ 157.63$$

**Try these**

A sum is taken for one year at 8% p. a. If interest is compounded after every three months, how many times will interest be charged in one year?

*Example 1.24*

Find the compound interest on ₹ 20,000 at 15% per annum for $2\frac{1}{3}$ years.

*Solution*

Here, P = ₹ 20,000, $r$ = 15% p. a. and $n = 2\frac{1}{3}$ years.

$$\begin{aligned}
\text{Amount after } 2\tfrac{1}{3} \text{ years} = A &= P\left(1+\frac{r}{100}\right)^2\left[1+\frac{1}{3}\left(\frac{r}{100}\right)\right] \\
&= 20000\left(1+\frac{15}{100}\right)^2\left[1+\frac{1}{3}\left(\frac{15}{100}\right)\right] \\
&= 20000\left(1+\frac{3}{20}\right)^2\left(1+\frac{1}{20}\right) \\
&= 20000\left(\frac{23}{20}\right)^2\left(\frac{21}{20}\right) \\
&= 20000\times\frac{23}{20}\times\frac{23}{20}\times\frac{21}{20} \\
&= ₹\ 27,772.50 \\
\text{C.I.} &= A - P \\
&= 27,772.50 - 20,000 \\
&= ₹\ 7,772.50
\end{aligned}$$

## Inverse Problems on Compound Interest

We have already learnt the formula, $A = P\left(1 + \frac{r}{100}\right)^n$,

where four variables A, P, $r$ and $n$ are involved. Out of these four variables, if any three variables are known, then we can calculate the fourth variable.

*Example 1.25*

At what rate per annum will ₹ 640 amount to ₹ 774.40 in 2 years, when interest is being compounded annually?

*Solution:*

**Given:** P = ₹ 640, A = ₹ 774.40, $n$ = 2 years, $r$ = ?

We know,

$$\begin{aligned}
A &= P\left(1 + \frac{r}{100}\right)^n \\
774.40 &= 640\left(1 + \frac{r}{100}\right)^2 \\
\frac{774.40}{640} &= \left(1 + \frac{r}{100}\right)^2 \\
\frac{77440}{64000} &= \left(1 + \frac{r}{100}\right)^2 \\
\frac{121}{100} &= \left(1 + \frac{r}{100}\right)^2 \\
\left(\frac{11}{10}\right)^2 &= \left(1 + \frac{r}{100}\right)^2 \\
\frac{11}{10} &= 1 + \frac{r}{100} \\
\frac{r}{100} &= \frac{11}{10} - 1 \\
\frac{r}{100} &= \frac{11-10}{10} \\
\frac{r}{100} &= \frac{1}{10} \\
r &= \frac{100}{10} \\
\text{Rate } r &= 10\% \text{ per annum.}
\end{aligned}$$

*Example 1.26*

In how much time will a sum of ₹ 1600 amount to ₹ 1852.20 at 5% per annum compound interest.

*Solution*

**Given:** P = ₹ 1600, A = ₹ 1852.20, $r$ = 5% per annum, $n$ = ?

We know,

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$1852.20 = 1600\left(1+\frac{5}{100}\right)^n$$

$$\frac{1852.20}{1600} = \left(\frac{105}{100}\right)^n$$

$$\frac{185220}{160000} = \left(\frac{21}{20}\right)^n$$

$$\frac{9261}{8000} = \left(\frac{21}{20}\right)^n$$

$$\left(\frac{21}{20}\right)^3 = \left(\frac{21}{20}\right)^n$$

$$\therefore n = 3 \text{ years}$$

**Try these**

Find the time period and rate for each of the cases given below :

1. A sum taken for 2 years at 8% p. a. compounded half - yearly.
2. A sum taken for 1½ years at 4% p. a. compounded half - yearly.

## 1.5 Difference between Simple Interest and Compound Interest

When P is the Principal, $n$ = 2 years and $r$ is the Rate of interest,

Difference between C. I. and S. I. for 2 years $= P\left(\frac{r}{100}\right)^2$

*Example 1.27*

Find the difference between Simple Interest and Compound Interest for a sum of ₹ 8,000 lent at 10% p. a. in 2 years.

*Solution*

Here, P = ₹ 8000, $n$ = 2 years, $r$ = 10% p. a.

Difference between Compound Interest and Simple Interest for 2 years $= P\left(\frac{r}{100}\right)^2$

$$= 8000\left(\frac{10}{100}\right)^2$$

$$= 8000\left(\frac{1}{10}\right)^2$$

$$= 8000 \times \frac{1}{10} \times \frac{1}{10} = ₹\ 80$$

### 1.5.1 Appreciation and Depreciation

**a) Appreciation**

In situations like growth of population, growth of bacteria, increase in the value of an asset, increase in price of certain valuable articles, etc., the following formula is used.

$$A = P\left(1 + \frac{r}{100}\right)^n$$

**b) Depreciation**

In certain cases where the cost of machines, vehicles, value of some articles, buildings, etc., decreases, the following formula can be used.

$$A = P\left(1 - \frac{r}{100}\right)^n$$

*Example 1.28*

The population of a village increases at the rate of 7% every year. If the present population is 90,000, what will be the population after 2 years?

*Solution*

Present population P = 90,000, Rate of increase $r = 7\%$,
Number of years $n = 2$.

The population after '$n$' years $= P\left(1 + \frac{r}{100}\right)^n$

$\therefore$ The population after two years $= 90000\left(1 + \frac{7}{100}\right)^2$

$= 90000\left(\frac{107}{100}\right)^2$

$= 90000 \times \frac{107}{100} \times \frac{107}{100}$

$= 103041$

The population after two years $=$ 1,03,041

### Example 1.29

The value of a machine depreciates by 5% each year. A man pays ₹ 30,000 for the machine. Find its value after three years.

**Solution**

Present value of the machine P = ₹ 30,000, Rate of depreciation $r = 5\%$,

$$\begin{aligned}
\text{Number of years } n &= 3 \\
\text{The value of the machine after '}n\text{' years} &= P\left(1-\frac{r}{100}\right)^n \\
\therefore \text{ The value of the machine after three years} &= 30000\left(1-\frac{5}{100}\right)^3 \\
&= 30000\left(\frac{95}{100}\right)^3 \\
&= 30000\times\frac{95}{100}\times\frac{95}{100}\times\frac{95}{100} \\
&= 25721.25 \\
\text{The value of the machine after three years} &= ₹\ 25{,}721.25
\end{aligned}$$

### Example 1.30

The population of a village has a constant growth of 5% every year. If its present population is 1,04,832, what was the population two years ago?

**Solution**

Let P be the population two years ago.

$$\begin{aligned}
\therefore P\left(1+\frac{5}{100}\right)^2 &= 104832 \\
P\left(\frac{105}{100}\right)^2 &= 104832 \\
P\times\frac{105}{100}\times\frac{105}{100} &= 104832 \\
P &= \frac{104832\times100\times100}{105\times105} \\
&= 95085.71 \\
&= 95{,}086 \text{ (rounding off to the nearest whole number)}
\end{aligned}$$

$\therefore$ Two years ago the population was 95,086.

**To find the formula for calculating interest and the maturity amount for R.D :**

Let $r$ % be the rate of interest paid and 'P' be the monthly instalment paid for '$n$' months.

$$\text{Interest} = \frac{\text{PN}r}{100}, \text{ where N} = \frac{1}{12}\left[\frac{n(n+1)}{2}\right] \text{ years}$$

$$\text{Total Amount due at maturity is A} = \text{P}n + \frac{\text{PN}r}{100}$$

*Example 1.31*

Tharun makes a deposit of Rupees two lakhs in a bank for 5 years. If the rate of interest is 8% per annum, find the maturity value.

**Solution**

Principal deposited P = ₹ 2,00,000, $n$ = 5 years, $r$ = 8% p. a.

$$\text{Interest} = \frac{\text{P}nr}{100} = 200000 \times 5 \times \frac{8}{100}$$

$$= ₹\ 80,000$$

$\therefore$ Maturity value after 5 years = 2,00,000 + 80,000 = ₹ 2,80,000.

*Example 1.32*

Vaideesh deposits ₹ 500 at the beginning of every month for 5 years in a post office. If the rate of interest is 7.5%, find the amount he will receive at the end of 5 years.

**Solution**

Amount deposited every month, P = ₹ 500

Number of months, $n$ = 5 × 12 = 60 months

Rate of interest, $r$ = $7\frac{1}{2}\% = \frac{15}{2}\%$

Total deposit made = P$n$ = 500 × 60

= ₹ 30,000

Period for recurring deposit, N = $\frac{1}{12}\left[\frac{n(n+1)}{2}\right]$ years

$$= \frac{1}{24} \times 60 \times 61 = \frac{305}{2} \text{ years}$$

$$\text{Interest, I} = \frac{PNr}{100}$$
$$= 500 \times \frac{305}{2} \times \frac{15}{2 \times 100}$$
$$= ₹\ 5{,}718.75$$
$$\text{Total amount due} = Pn + \frac{PNr}{100}$$
$$= 30{,}000 + 5{,}718.75$$
$$= ₹\ 35{,}718.75$$

*Example 1.33*

Vishal deposited ₹ 200 per month for 5 years in a recurring deposit account in a post office. If he received ₹ 13,830 find the rate of interest.

*Solution*

Maturity Amount, A = ₹ 13,830, P = ₹ 200, $n = 5 \times 12 = 60$ months

$$\text{Period, N} = \frac{1}{12}\left[\frac{n(n+1)}{2}\right] \text{ years}$$
$$= \frac{1}{12} \times 60 \times \frac{61}{2} = \frac{305}{2} \text{ years}$$
$$\text{Amount Deposited} = Pn = 200 \times 60 = ₹\ 12,\ 000$$
$$\text{Maturity Amount} = Pn + \frac{PNr}{100}$$
$$13830 = 12000 + 200 \times \frac{305}{2} \times \frac{r}{100}$$
$$13830 - 12000 = 305 \times r$$
$$1830 = 305 \times r$$
$$\therefore r = \frac{1830}{305} = 6\%$$

### Equated Monthly Instalment ( E.M.I. )

Equated Monthly Instalment is also as equivalent as the instalment scheme but with a dimnishing concept. We have to repay the cost of things with the interest along with certain charges. The total amount should be divided by the period of months. The amount thus arrived is known as Equated Monthly Instalment.

$$\text{E.M.I} = \frac{\text{Principal} + \text{Interest}}{\text{Number of months}}$$

### Different schemes of Hire purchase and Instalment scheme

**1. 0% interest scheme:** Companies take processing charge and 4 or 5 months instalments in advance.

**2. 100% Finance:** Companies add interest and the processing charges to the cost price.

**3. Discount Sale:** To promote sales, discount is given in the instalment schemes.

**4. Initial Payment:** A certain part of the price of the article is paid towards the purchase in advance. It is also known as Cash down payment.

## 1.7 Compound Variation

In the earlier classes we have already learnt about Direct and Inverse Variation. Let us recall them.

### Direct Variation

If two quantities are such that an increase or decrease in one leads to a corresponding increase or decrease in the other, we say they vary directly or the variation is Direct.

### Examples for Direct Variation:

1. Distance and Time are in Direct Variation, because more the distance travelled, the time taken will be more( if speed remains the same).
2. Principal and Interest are in Direct Variation, because if the Principal is more the interest earned will also be more.
3. Purchase of Articles and the amount spent are in Direct Variation, because purchase of more articles will cost more money.

## Indirect Variation or Inverse Variation:

If two quantities are such that an increase or decrease in one leads to a corresponding decrease or increase in the other, we say they vary indirectly or the variation is inverse.

## Examples for Indirect Variation:

1. Work and time are in Inverse Variation, because more the number of the workers, lesser will be the time required to complete a job.
2. Speed and time are in Inverse Variation, because higher the speed, the lower is the time taken to cover a distance.
3. Population and quantity of food are in Inverse Variation, because if the population increases the food availability decreases.

## Compound Variation

Certain problems involve a chain of two or more variations, which is called as Compound Variation.

The different possibilities of variations involving two variations are shown in the following table:

| Variation I | Variation II |
|---|---|
| Direct | Direct |
| Inverse | Inverse |
| Direct | Inverse |
| Inverse | Direct |

Let us work out some problems to illustrate compound variation.

*Example 1.36*

If 20 men can build a wall 112 meters long in 6 days, what length of a similar wall can be built by 25 men in 3 days?

*Solution:*

**Method 1:** The problem involves set of 3 variables, namely- Number of men, Number of days and length of the wall.

| Number of Men | Number of days | Length of the wall in metres |
|---|---|---|
| 20 | 6 | 112 |
| 25 | 3 | $x$ |

Therefore, the proportion is 20 : 25 :: 112 : $x$ ...... (1)

**Step 2:** Consider the number of days and the length of the wall. As the number of days decreases from 6 to 3, the length of the wall also decreases. So, it is in Direct Variation.

Therefore, the proportion is 6 : 3 :: 112 : $x$ ..... (2)

Combining (1) and (2), we can write

$$\left.\begin{array}{r}20:25\\ 6:3\end{array}\right\}::112:x$$

We know, **Product of Extremes = Product of Means.**

| Extremes | | Means | | Extremes |
|---|---|---|---|---|
| 20 | : | 25 ::112 | : | $x$ |
| 6 | : | 3 | | |

So, $20 \times 6 \times x = 25 \times 3 \times 112$

$$x = \frac{25 \times 3 \times 112}{20 \times 6} = 70 \text{ meters.}$$

**Method 2**

| Number of Men | Number of days | Length of the wall in metres |
|---|---|---|
| 20 | 6 | 112 |
| 25 | 3 | $x$ |

**Step 1:** Consider the number of men and length of the wall. As the number of men increases from 20 to 25, the length of the wall also increases. It is in direct variation.

$$\text{The multiplying factor} = \frac{25}{20}$$

**Step 2:** Consider the number of days and the length of the wall. As the number of days decreases from 6 to 3, the length of the wall also decreases. It is in direct variation.

$$\text{The multiplying factor} = \frac{3}{6}.$$

$$\therefore x = \frac{25}{20} \times \frac{3}{6} \times 112 = 70 \text{ meters}$$

*Example 1.37*

Six men working 10 hours a day can do a piece of work in 24 days. In how many days will 9 men working for 8 hours a day do the same work?

*Solution*

**Method 1:** The problem involves 3 sets of variables, namely - Number of men, Working hours per day and Number of days.

| Number of Men | Number of hours per day | Number of days |
|---|---|---|
| 6 | 10 | 24 |
| 9 | 8 | $x$ |

**Step 1:** Consider the number of men and the number of days. As the number of men increases from 6 to 9, the number of days decreases. So it is in Inverse Variation.

Therefore the proportion is $9 : 6 :: 24 : x$ ..... (1)

**Step 2:** Consider the number of hours worked per day and the number of days. As the number of hours working per day decreases from 10 to 8, the number of days increases. So it is inverse variation.

Therefore the proportion is $8 : 10 :: 24 : x$ ..... (2)

Combining (1) and (2), we can write as

$$\left.\begin{matrix} 9:6 \\ 8:10 \end{matrix}\right\} :: 24 : x$$

We know, **Product of extremes = Product of Means.**

| Extremes | | Means | | Extremes |
|---|---|---|---|---|
| 9 | : | 6 :: 24 | : | $x$ |
| 8 | : | 10 | | |

So, $9 \times 8 \times x = 6 \times 10 \times 24$

$$x = \frac{6 \times 10 \times 24}{9 \times 8} = 20 \text{ days}$$

**Note:** 1. Denote the Direct variation as ↓ (Downward arrow)
2. Denote the Indirect variation as ↑ (Upward arrow)
3. Multiplying Factors can be written based on the arrows. Take the number on the head of the arrow in the numerator and the number on the tail of the arrow in the denominator.

For method two, use the instructions given in the note above .

**Method 2 : (Using arrow marks)**

| Number of Men | Number of hours per day | Number of days |
|---|---|---|
| 6 | 10 | 24 |
| 9 | 8 | $x$ |

**Step 1 :** Consider men and days. As the number of men increases from 6 to 9, the number of days decreases. It is in inverse variation.

The multiplying factor = $\frac{6}{9}$

**Step 2 :** Consider the number of hours per day and the number of days. As the number of hours per day decreases from 10 to 8, the number of days increases. It is also in inverse variation.

The multiplying factor = $\frac{10}{8}$

$$\therefore x = \frac{6}{9} \times \frac{10}{8} \times 24 = 20 \text{ days.}$$

## 1.8 Time and Work

When we have to compare the work of several persons, it is necessary to ascertain the amount of work each person can complete in one day. As time and work are of inverse variation and if more people are joined to do a work, the work will be completed within a shorter time.

In solving problems here, the following points should be remembered:

1. If a man finishes total work in '$n$' days, then in one day he does '$\frac{1}{n}$' of the total work. For example, if a man finishes a work in 4 days, then in one day he does $\frac{1}{4}$ of the work.
2. If the quantity of work done by a man in one day is given, then the total number of days taken to finish the work = 1/(one day's work). For example, if a man does $\frac{1}{10}$ of the work in 1 day, then the number of days taken to finish the work

$$= \frac{1}{\left(\frac{1}{10}\right)} = 1 \times \frac{10}{1} = 10 \text{ days.}$$

*Example 1.38*

A can do a piece of work in 20 days and B can do it in 30 days. How long will they take to do the work together?

*Solution*

Work done by A in 1 day = $\frac{1}{20}$, Work done by B in 1 day = $\frac{1}{30}$

$$\text{Work done by A and B in 1 day} = \frac{1}{20} + \frac{1}{30}$$

$$= \frac{3+2}{60} = \frac{5}{60} = \frac{1}{12} \text{ of the work}$$

Total number of days required to finish the work by A and B = $\frac{1}{1/12} = 12$ days.

*Example 1.39*

A and B together can do a piece of work in 8 days, but A alone can do it 12 days. How many days would B alone take to do the same work?

*Solution*

$$\text{Work done by A and B together in 1 day} = \frac{1}{8} \text{ of the work}$$

$$\text{Work done by A in 1 day} = \frac{1}{12} \text{ of the work}$$

$$\text{Work done by B in 1 day} = \frac{1}{8} - \frac{1}{12} = \frac{3-2}{24} = \frac{1}{24}$$

Number of days taken by B alone to do the same work = $\frac{1}{1/24} = 24$ days.

*Example 1.40*

Two persons A and B are engaged in a work. A can do a piece of work in 12 days and B can do the same work in 20 days. They work together for 3 days and then A goes away. In how many days will B finish the work?

**Try these**

While A, B and C working individually can complete a job in 20,5,4 days respectively. If all join together and work, find in how many days they will finish the job?

**Solution**

$$\text{Work done by A in 1 day} = \frac{1}{12}$$

$$\text{Work done by B in 1 day} = \frac{1}{20}$$

$$\text{Work done by A and B together in 1 day} = \frac{1}{12}+\frac{1}{20}$$

$$= \frac{5+3}{60} = \frac{8}{60} = \frac{2}{15}$$

$$\text{Work done by A and B together in 3 days} = \frac{2}{15}\times 3 = \frac{2}{5}$$

$$\text{Remaining Work} = 1-\frac{2}{5} = \frac{3}{5}$$

$$\text{Number of days taken by B to finish the remaining work} = \frac{\frac{3}{5}}{\frac{1}{20}} = \frac{3}{5}\times\frac{20}{1}$$

$$= 12 \text{ days.}$$

*Example 1.41*

A and B can do a piece of work in 12 days, B and C in 15 days, C and A in 20 days. In how many days will they finish it together and separately?

**Solution**

$$\text{Work done by A and B in 1 day} = \frac{1}{12}$$

$$\text{Work done by B and C in 1 day} = \frac{1}{15}$$

$$\text{Work done by C and A in 1 day} = \frac{1}{20}$$

$$\text{Work done by (A+B)+(B+C)+(C+A) in 1 day} = \frac{1}{12}+\frac{1}{15}+\frac{1}{20}$$

$$\text{Work done by } (2A+2B+2C) \text{ in 1 day} = \frac{5+4+3}{60}$$

$$\text{Work done by } 2(A+B+C) \text{ in 1 day} = \frac{12}{60}$$

$$\text{Work done by A, B and C together in 1 day} = \frac{1}{2} \times \frac{12}{60} = \frac{1}{10}$$

$\therefore$ A,B and C will finish the work in 10 days.

Work done by A in 1 day

(i.e.) $[(A+B+C)\text{'s work} - (B+C)\text{'s work}] = \frac{1}{10} - \frac{1}{15} = \frac{3-2}{30} = \frac{1}{30}$

$\therefore$ A will finish the work in 30 days.

Workdone by B in 1 day

(i.e.) $[(A+B+C)\text{'s work} - (C+A)\text{'s work}] = \frac{1}{10} - \frac{1}{20} = \frac{2-1}{20} = \frac{1}{20}$

$\therefore$ B will finish the work in 20 days.

Work done by C in 1 day

(i.e.) $[(A+B+C)\text{'s work} - (A+B)\text{'s work}] = \frac{1}{10} - \frac{1}{12} = \frac{6-5}{60} = \frac{1}{60}$

$\therefore$ C will finish the work in 60 days.

***Example 1.42***

A can do a piece of work in 10 days and B can do it in 15 days. How much does each of them get if they finish the work and earn ₹ 1500?

***Solution***

$$\text{Work done by A in 1 day} = \frac{1}{10}$$

$$\text{Work done by B in 1 day} = \frac{1}{15}$$

$$\text{Ratio of their work} = \frac{1}{10} : \frac{1}{15} = 3 : 2$$

$$\text{Total Share} = ₹\ 1500$$

$$\text{A's share} = \frac{3}{5} \times 1500 = ₹\ 900$$

$$\text{B's share} = \frac{2}{5} \times 1500 = ₹\ 600$$

*Example 1.43*

Two taps can fill a tank in 30 minutes and 40 minutes. Another tap can empty it in 24 minutes. If the tank is empty and all the three taps are kept open, in how much time the tank will be filled?

**Solution**

Quantity of water filled by the first tap in one minute $= \frac{1}{30}$

Quantity of water filled by the second tap in one minute $= \frac{1}{40}$

Quantity of water emptied by the third tap in one minute $= \frac{1}{24}$

Quantity of water filled in one minute, when all the 3 taps are opened $\Big\} = \frac{1}{30} + \frac{1}{40} - \frac{1}{24}$

$$= \frac{4+3-5}{120} = \frac{7-5}{120}$$

$$= \frac{2}{120} = \frac{1}{60}$$

Time taken to fill the tank $= \frac{1}{1/60} = 60$ minutes

$= 1$ hour

**Abbreviation:** C.P. = Cost Price, S.P. = Selling Price, M.P. = Marked Price,

P = Principal, $r$ = Rate of interest, $n$ = time period,

A = Amount, C. I. = Compound Interest.

- Percent means per hundred. A fraction with its denominator 100 is called a percent.
- In case of profit, we have

  Profit $= S.P. - C.P.$ ; Profit percent $= \frac{\text{Profit}}{C.P} \times 100$

  $S.P. = \left(\frac{100 + \text{Profit\%}}{100}\right) \times C.P.$; $C.P. = \left(\frac{100}{100 + \text{Profit\%}}\right) \times S.P.$
- In case of Loss, we have

  Loss $= C.P. - S.P.$; Loss percent $= \frac{\text{Loss}}{C.P.} \times 100$

  $S.P. = \left(\frac{100 - \text{Loss\%}}{100}\right) \times C.P.$; $C.P. = \left(\frac{100}{100 - \text{Loss\%}}\right) \times S.P.$
- Discount is the reduction given on the Marked Price.
- Selling Price is the price payable after reducing the Discount from the Marked Price.
- Discount $= M.P. - S.P.$
- $M.P. = \frac{100}{100 - \text{Discount\%}} \times S.P.$; $S.P. = \frac{100 - \text{Discount\%}}{100} \times M.P.$
- $C.P. = \frac{100 - \text{Discount\%}}{100 + \text{Profit\%}} \times M.P.$; $M.P. = \frac{100 + \text{Profit\%}}{100 - \text{Discount\%}} \times C.P.$
- Discount Percent $= \frac{\text{Discount}}{M.P.} \times 100.$
- When the interest is

  (i) compounded annually, $A = P\left(1 + \frac{r}{100}\right)^n$

  (ii) compounded half - yearly, $A = P\left[1 + \frac{1}{2}\left(\frac{r}{100}\right)\right]^{2n}$

  (iii) compounded quarterly, $A = P\left[1 + \frac{1}{4}\left(\frac{r}{100}\right)\right]^{4n}$
- Appreciation, $A = P\left(1 + \frac{r}{100}\right)^n$; Depreciation, $A = P\left(1 - \frac{r}{100}\right)^n$
- The difference between C. I. and S. I. for 2 years $= P\left(\frac{r}{100}\right)^2$
- One day's work of A $= \frac{1}{\text{Number of days taken by A}}$
- Work completed in '$x$' days = One day's work x $x$

## Pythagorean Triplets.

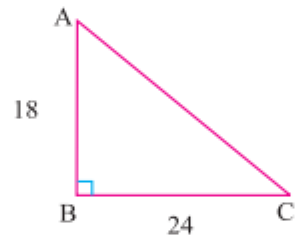
### *Example 2.1*

In $\Delta ABC, \angle B = 90^\circ$, AB = 18cm and BC = 24cm. Calculate the length of AC.

***Solution***

By Pythagoras Theorem,

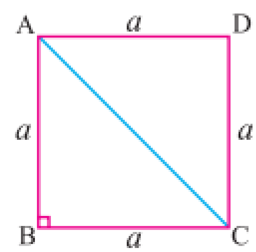
$$
\begin{aligned}
AC^2 &= AB^2 + BC^2 \\
&= 18^2 + 24^2 \\
&= 324 + 576 \\
&= 900 \\
\therefore AC &= \sqrt{900} = 30\text{cm}
\end{aligned}
$$




### *Example 2.2*

A square has the perimeter 40cm. What is the sum of the diagonals?

***Solution***




Let '$a$' be the length of the side of the square. AC is a diagonal.

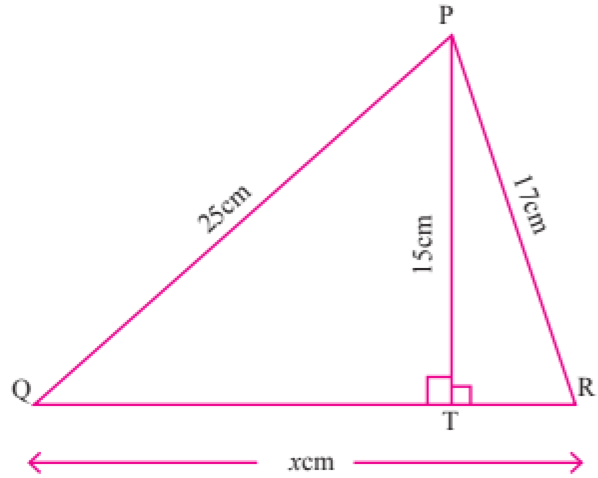
$$
\begin{aligned}
\text{Perimeter of square ABCD} &= 4\,a \text{ units} \\
4a &= 40\text{cm [given]} \\
a &= \frac{40}{4} = 10\text{cm}
\end{aligned}
$$

We know that in a square each angle is $90^\circ$ and the diagonals are equal.

$$
\begin{aligned}
\text{In } \Delta ABC,\ AC^2 &= AB^2 + BC^2 \\
&= 10^2 + 10^2 = 100 + 100 = 200 \\
\therefore AC &= \sqrt{200} \\
&= \sqrt{2 \times 100} = 10\sqrt{2} \\
&= 10 \times 1.414 = 14.14\text{cm} \\
\text{Diagonal AC} &= \text{Diagonal BD} \\
\text{Hence, Sum of the diagonals} &= 14.14 + 14.14 = 28.28\text{cm.}
\end{aligned}
$$

*Example 2.3*

From the figure PT is an altitude of the triangle PQR in which PQ = 25cm, PR = 17cm and PT = 15 cm. If QR = $x$ cm. Calculate $x$.




***Solution*** From the figure, we have QR = QT + TR.

To find : QT and TR.

In the right angled triangle PTQ , $\angle PTQ = 90^\circ$ [PT is attitude]

By Pythagoras Theorem, $PQ^2 = PT^2 + QT^2$ $\quad \therefore PQ^2 - PT^2 = QT^2$

$$\therefore QT^2 = 25^2 - 15^2 = 625 - 225 = 400$$

$$QT = \sqrt{400} = 20 \text{ cm} \qquad .....(1)$$

Similarly, in the right angled triangle PTR,

by Pythagoras Theorem, $PR^2 = PT^2 + TR^2$

$$\therefore TR^2 = PR^2 - PT^2$$
$$= 17^2 - 15^2$$
$$= 289 - 225 = 64$$
$$TR = \sqrt{64} = 8 \text{ cm} \qquad ..... (2)$$

Form (1) and (2) $\quad QR = QT + TR = 20 + 8 = 28\text{cm}.$

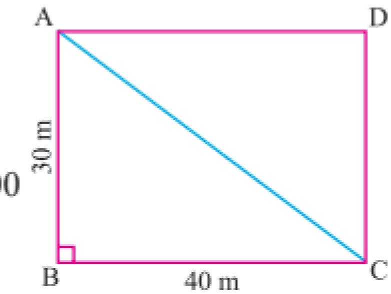
*Example 2.4*

A rectangular field is of dimension 40m by 30m. What distance is saved by walking diagonally across the field?

*Solution*

Given: ABCD is a rectangular field of Length = 40m, Breadth = 30m, $\angle B = 90^\circ$

In the right angled triangle ABC,




By Pythagoras Theorem,

$$AC^2 = AB^2 + BC^2$$
$$= 30^2 + 40^2 = 900 + 1600$$
$$= 2500$$
$$\therefore AC = \sqrt{2500} = 50 \text{ m}$$

Distance from A to C through B is

$$= 30 + 40 = 70 \text{ m}$$

$$\text{Distance saved} = 70 - 50 = 20 \text{ m.}$$

**Diameter**

**A diameter is a chord that passes through the centre of the circle and diameter is the longest chord of a circle.**

In the figure, AOB is diameter of the circle.

O is the mid point of AB and OA= OB = radius of the circle

Hence, Diameter = 2 × radius (or) Radius = ( diameter ) ÷ 2

**Note :** (i) The mid-point of every diameter of the circle is the centre of the circle.

(ii) The diameters of a circle are concurrent and the point of concurrency is the centre of the circle.

- **Centroid :** Point of concurrency of the three Medians.
- **Orthocentre :** Point of concurrency of the three Altitudes.
- **Incentre :** Point of concurrency of the three Angle Bisectors.
- **Circumcentre :** Point of concurrency of the Perpendicular Bisectors of the three sides.
- **Circle :** A circle is the set of all points in a plane at a constant distance from a fixed point in that plane .
- **Chord :** A chord is a line segment with its end points lying on a circle.
- **Diameter :** A diameter is a chord that passes through the centre of the circle.
- A line passing through a circle and intersecting the circle at two points is called the secant of the circle.
- Tangent is a line that touches a circle at exactly one point, and the point is known as point of contact.
- **Segment of a circle :** A chord of a circle divides the circular region into two parts.
- **Sector of a circle :** The circular region enclosed by an arc of a circle and the two radii at its end points is known as Sector of a circle.

## 3.5 Measures of Central Tendency

Even after tabulating the collected mass of data, we get only a hazy general picture of the distribution. To obtain a more clear picture, it would be ideal if we can describe the whole mass of data by a single number or representative number. To get more information about the tendency of the data to deviate about a particular value, there are certain measures which characterise the entire data. These measures are called the **Measures of Central Tendency**. Such measures are

(i) Arithmetic Mean, (ii) Median and (iii) Mode

### 3.5.1 Arithmetic Mean (A.M)

The arithmetic mean is the ratio of the sum of all the observations to the total number of observations.

### 3.5.1. (a) Arithmetic mean for ungrouped data

If there are $n$ observations $x_1, x_2, x_3, \cdots, x_n$ for the variable $x$ then their arithmetic mean is denoted by $\overline{x}$ and it is given by $\overline{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$.

In Mathematics, the symbol in Greek letter $\Sigma$, is called **Sigma.** This notation is used to represent the summation. With this symbol, the sum of $x_1, x_2, x_3, \cdots, x_n$ is denoted as $\sum_{i=1}^{n} x_i$ or simply as $\Sigma x_i$. Then we have $\overline{x} = \frac{\Sigma x_i}{n}$.

**Note:** Arithmetic mean is also known as **Average** or **Mean**.

**More about Notation : $\Sigma$**

$$\sum_{k=1}^{3} k = 1 + 2 + 3 = 6$$

$$\sum_{n=3}^{6} n = 3 + 4 + 5 + 6 = 18$$

$$\sum_{n=2}^{4} 2n = 2 \times 2 + 2 \times 3 + 2 \times 4 = 18$$

$$\sum_{k=1}^{3} 5 = \sum_{k=1}^{3} 5 \times k^0$$
$$= 5 \times 1^0 + 5 \times 2^0 + 5 \times 3^0$$
$$= 5 + 5 + 5 = 15$$

$$\sum_{K=2}^{4} (k-1) = (2-1) + (3-1) + (4-1) = 6$$

*Example 3.12*

The marks obtained by 10 students in a test are 15, 75, 33, 67, 76, 54, 39, 12, 78, 11. Find the arithmetic mean.

*Solution*

Here, the number of observations, $n = 10$

$$\text{A. M} = \overline{x} = \frac{15 + 75 + 33 + 67 + 76 + 54 + 39 + 12 + 78 + 11}{10}$$

$$\overline{x} = \frac{460}{10} = 46.$$

*Example 3.13*

If the average of the values 9, 6, 7, 8, 5 and $x$ is 8. Find the value of $x$.

*Solution*

Here, the given values are 9, 6, 7, 8, 5 and $x$, also $n = 6$.

$$\text{By formula, A.M.} = \overline{x} = \frac{9 + 6 + 7 + 8 + 5 + x}{6} = \frac{35 + x}{6}$$

$$\text{By data, } \overline{x} = 8$$

$$\text{So, } \frac{35 + x}{6} = 8$$

$$\text{i.e. } 35 + x = 48$$

$$x = 48 - 35 = 13.$$

### *Example 3.14*

The average height of 10 students in a class was calculated as 166 cm. On verification it was found that one reading was wrongly recorded as 160 cm instead of 150 cm. Find the correct mean height.

**Solution**

$$\text{Here, } \overline{x} = 166 \text{ cm and } n = 10$$

$$\text{We have } \overline{x} = \frac{\Sigma x}{n} = \frac{\Sigma x}{10}$$

$$\text{i.e. } 166 = \frac{\Sigma x}{10} \text{ or } \Sigma x = 1660$$

$$\text{The incorrect } \Sigma x = 1660$$

$$\text{The correct } \Sigma x = \text{incorrect } \Sigma x - \text{the wrong value} + \text{correct value}$$

$$= 1660 - 160 + 150 = 1650$$

$$\text{Hence, the correct A.M.} = \frac{1650}{10} = 165 \text{ cm.}$$

### *Example 3.15*

Calculate the Arithmetic mean of the following data by direct method

| $x$ | 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|
| $f$ | 4 | 5 | 7 | 4 | 3 | 2 |

**Solution**

| $x$ | $f$ | $fx$ |
|---|---|---|
| 5 | 4 | 20 |
| 10 | 5 | 50 |
| 15 | 7 | 105 |
| 20 | 4 | 80 |
| 25 | 3 | 75 |
| 30 | 2 | 60 |
| Total | N = 25 | $\Sigma fx = 390$ |

$$\text{Arithmetic Mean, } \overline{x} = \frac{\Sigma fx}{N}$$

$$= \frac{390}{25} = 15.6\,.$$

### 3.5.3 Median

Another measure of central tendency is the **Median**.

### 3.5.3 (a) To find Median for ungrouped data

The median is calculated as follows:

(i) Suppose there are an **odd number** of observations, write them in ascending or descending order. Then the middle term is the Median.

For example: Consider the five observations 33, 35, 39, 40, 43. The middle most value of these observation is 39. It is the Median of these observation.

(ii) Suppose there are an **even number** of observations, write them in ascending or descending order. Then the average of the two middle terms is the Median.

For example, the median of 33, 35, 39, 40, 43, 48 is $\frac{39+40}{2} = 39.5$.

*Example 3.17*

Find the median of 17, 15, 9, 13, 21, 7, 32.

***Solution***

Arrange the values in the ascending order as 7, 9, 13, 15, 17, 21,32,

$$\text{Here, } n = 7 \text{ (odd number)}$$

$$\text{Therefore, Median} = \text{Middle value}$$

$$= \left(\frac{n+1}{2}\right)^{th} \text{value} = \left(\frac{7+1}{2}\right)^{th} \text{value} = 4\text{th value.}$$

Hence, the median is 15.

*Example 3.18*

A cricket player has taken the runs 13, 28, 61, 70, 4, 11, 33, 0, 71, 92. Find the median.

***Solution***

Arrange the runs in ascending order as 0, 4, 11, 13, 28, 33, 61, 70, 71, 92.

Here $n = 10$ (even number).

There are two middle values 28 and 33.

$$\therefore \text{Median} = \text{Average of the two middle values}$$

$$= \frac{28+33}{2} = \frac{61}{2} = 30.5 .$$

### 3.5.3 (b) To find Median for grouped data

### Cumulative frequency

**Cumulative frequency** of a class is nothing but the total frequency upto that class.

*Example 3.19*

Find the median for marks of 50 students

| Marks | 20 | 27 | 34 | 43 | 58 | 65 | 89 |
|---|---|---|---|---|---|---|---|
| Number of students | 2 | 4 | 6 | 11 | 12 | 8 | 7 |

*Solution*

| Marks ($x$) | Number of students ($f$) | Cumulative frequency |
|---|---|---|
| 20 | 2 | 2 |
| 27 | 4 | $(2+4=)$ 6 |
| 34 | 6 | $(6+6=)$ 12 |
| 43 | 11 | $(11+12=)$ 23 |
| **58** | **12** | **$(23+12=)$ 35** |
| 65 | 8 | $(35+8=)$ 43 |
| 89 | 7 | $(43+7=)$ 50 |

Here, the total frequency, $N = \Sigma f = 50$

$$\therefore \frac{N}{2} = \frac{50}{2} = 25.$$

The median is $\left(\frac{N}{2}\right)^{th}$ value $=$ 25th value.

Now, **25th value occurs in the cummulative frequency 35, whose corresponding marks is 58.**

Hence, the median = 58.

### 3.5.4 Mode

**Mode** is also a measure of central tendency.

The Mode can be calculated as follows:

### 3.5.4 (a) To find Mode for ungrouped data (Discrete data)

If a set of individual observations are given, then the Mode is the value which occurs most often.

***Example 3.20***

Find the mode of 2, 4, 5, 2, 1, 2, 3, 4, 4, 6, 2.

***Solution***

In the above example the number 2 occurs maximum number of times.

ie, 4 times. Hence mode = 2.

***Example 3.21***

Find the mode of 22, 25, 21, 22, 29, 25, 34, 37, 30, 22, 29, 25.

***Solution***

Here 22 occurs 3 times and 25 also occurs 3 times

$\therefore$ Both 22 and 25 are the modes for this data. We observe that there are two modes for the given data.

*Example 3.22*

Find the mode of 15, 25, 35, 45, 55, 65,

*Solution*

Each value occurs exactly one time in the series. Hence there is no mode for this data.

### 3.5.4 (b) To find Mode for grouped data (Frequency distribution)

If the data are arranged in the form of a frequency table, the class corresponding to the maximum frequency is called the modal class. The value of the variate of the modal class is the **mode**.

*Example: 3.23*

Find the mode for the following frequency table

| Wages (₹) | 250 | 300 | 350 | 400 | 450 | 500 |
|---|---|---|---|---|---|---|
| Number of workers | 10 | 15 | 16 | 12 | 11 | 13 |

*Solution*

| Wages (₹) | Number of workers |
|---|---|
| 250 | 10 |
| 300 | 15 |
| **350** | **16** |
| 400 | 12 |
| 450 | 11 |
| 500 | 13 |

We observe from the above table that the maximum frequency is 16. **The value of the variate (wage) corresponding to the maximum frequency 16 is 350.** This is the mode of the given data.